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MANAGING CATASTROPHIC RISK BY ALTERNATIVE RISK TRANSFER INSTRUMENTS

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MANAGING CATASTROPHIC RISK BY ALTERNATIVE RISK TRANSFER INSTRUMENTS

Abstract

Chapter 1 analyzes hybrid-trigger CAT bonds, a new CAT bond deal that can reduce basis risk and eliminate moral hazard simultaneously. It is the first research that provides analytical evidence on the condition under which the hybrid trigger has lower basis risk. Simulation results support my analyses. Major findings in this study provide insights to insurers who would proactively manage the basis risk of CAT bonds. Chapter 2 examines whether the parimutuel mechanism can hedge risk-averse people against catastrophic losses. Two optimal stake choice models are constructed. In the first model where the stakes of other players are exogenous, the optimal stake can be obtained by equating the marginal cost of a net payoff with the ratio of the expected marginal utilities in the payoff state and the no payoff state. The dynamic optimal hedge can be achieved if the odds, and the conditional probability of a hurricane hitting the target area, are available. In the second model, an optimal equilibrium stake is derived by maximizing the representative agent's expected utility. Given no transaction fee and tax, we show that parimutuel insurance intrinsically leads to participants being underinsured due to basis risk. Although participants will be underinsured, parimutuel insurance guarantees no underlying risk borne by the issuer. We also derive the equivalent transaction costs of traditional insurance relative to HuRLOs. The actual transaction cost for traditional insurance is found to be higher than the equivalent utility level implied by HuRLOs, suggesting that hedgers would be better off with HuRLOs than with traditional insurance. Chapter 3 analyzes the implications of climate change for catastrophic risk and examines the appropriateness of longer term insurance contracts to protect insurers against catastrophic losses and changes in risk estimates over time. Climate change essentially plays an important role in modeling catastrophic risks, especially in the tail of the loss distribution and for longer time scales. Mitigations can completely offset the impact of climate change. Longer term insurance contract may stimulate the incentive to invest on mitigation; however, risk capital required and annual premiums could increase significantly due to the additional premium risk faced by the insurers.

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Second Advisor

Neil A. Doherty

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Gregory Nini

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MANAGING CATASTROPHIC RISK BY ALTERNATIVE RISK TRANSFER
INSTRUMENTS

Chieh Ou Yang

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ABSTRACT

MANAGING CATASTROPHIC RISK BY ALTERNATIVE RISK TRANSFER

INSTRUMENTS

Chieh Ou Yang

Howard Kunreuther

Chapter 1 analyzes hybrid-trigger CAT bonds, a new CAT bond deal that can reduce basis risk and eliminate moral hazard simultaneously. It is the first research that provides analytical evidence on the condition under which the hybrid trigger has lower basis risk. Simulation results support my analyses. Major findings in this study provide insights to insurers who would proactively manage the basis risk of CAT bonds. Chapter 2 examines whether the parimutuel mechanism can hedge risk-averse people against catastrophic losses. Two optimal stake choice models are constructed. In the first model where the stakes of other players are exogenous, the optimal stake can be obtained by equating the marginal cost of a net payoff with the ratio of the expected marginal utilities in the payoff state and the no payoff state. The dynamic optimal hedge can be achieved if the odds, and the conditional probability of a hurricane hitting the target area, are available. In the second model, an optimal equilibrium stake is derived by maximizing the representative agent's expected utility. Given no transaction fee and tax, we show that parimutuel insurance intrinsically leads to participants being underinsured due to basis risk. Although participants will be underinsured, parimutuel insurance guarantees no underlying risk borne by the issuer. We also derive the equivalent transaction costs of traditional insurance relative to HuRLOs. The actual transaction cost for traditional

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General Introduction

Recent statistics of economic and insured losses caused by great natural disasters allow us to witness the rising pattern of hazard exposure to catastrophes. Figure 1 depicts economic and insured losses from natural disasters from 1950 to 2008 throughout the world. We can observe that the losses caused by great natural disasters increase dramatically in recent years, especially after 1990. Although the losses in 2000-2003 and 2006-2007 were less severe than previous years, those in 2004, 2005, and 2008 picked up again. Similarly, catastrophes also have an escalated impact on insurers over the last 15 years than in the entire history of insurance, as revealed in Figure 2. Before 1988, the worldwide insured losses from natural disasters are at most approximately \$10 billion dollars. In contrast, after 1990, insured losses rise significantly. Insured losses increase up to \$85 billion dollars when hurricane Katrina, Wilma, and Rita hit Gulf of Mexico in 2005. Natural disasters not only take away lives of human but also have adverse impacts on economy by causing direct damage, such as property destruction, or indirect damage, such as business interruption and production losses. Representative extreme events in recent years demonstrated the nature of diversity and globalization on natural disasters. Eastern Coast and Gulf of Mexico in U.S. stroke by a series of hurricanes. In 2005, Hurricane Katrina inflicted \$46.3 billion in insured losses and generated more than \$100 billion in federal aid. Hurricane Ike cost insurers 16 billion losses in 2008. In Europe, Germany suffered from Elbe floods in 2002; the 2003 heat wave caused 35,000 deaths; river floods annoyed UK in 2007; Greece was threatened by wildfires in 2008. In Asia,

tsunami in 2004 caused more than 283,000 deaths; a destructive earthquake in China killed 70,000 people in 2008.

Increased storm activities and rapid coastal development are the major sources that greatly elevate the risk of economic losses from natural disasters. Recent literature from scientific community suggests that global warming, primarily from human contributions, is likely to be responsible for changes in weather patterns, including the number, severity, and paths of storms and rainfall quantity, contributing to greater frequency and intensity of natural catastrophes¹. Based on a study by National Oceanic & Atmospheric Administration (NOAA), in the most recent decade, more people moved to coastal area, especially the Atlantic coasts and Gulf of Mexico where hurricane threat is the greatest². Although scientists seem to have different opinions in the assessments of the level and the nature of weather-related risk in coastal areas, the risk has been generally believed to increase worldwide. As indicated, economic and insured losses have increased substantially over the last 15 years, consistent with the elevated storm activities and the recent coastal development.

¹ Emanuel (2005) exhibits that more intensified long-lived storms caused escalating power dissipation since mid-1970s, which in turn stimulates more destructive hurricanes. Webster et al. (2005) also provide evidence on an elevated pattern on frequency and severity of hurricanes in recent 30 years. Based on Hoyos et al. (2006), the substantially enhanced hurricane intensity may attribute to the rise in sea surface temperature, which is related to global warming. Human behaviors, including the burning of fossil fuels, deforestation, and other land use changes, contribute to the emission of carbon dioxide and other greenhouse gasses, such as methane, which have accumulated in the atmosphere since late 19th century. Greenhouse gasses trap heat more easily, resulting in higher surface air temperature. IPCC predicts that global average surface temperatures will increase 1.1°C~2.9°C under a low emission scenario and 2.4°C~6.4°C under a high emission scenario. Stern (2007) also suggests that positive feedback mechanisms of climate change, such as releases of methane resulting from melting of permafrost and a reduced uptake of carbon that caused by shrinking Amazon forest, may amplify greenhouse gas concentrations and lead to global warming that is more severe than anticipated by climate models.

² During 1980 to 2003, 33 million residents are added to populations in coastal areas, posed an overall 28 percentage increase, but the growth rate is higher in southern Atlantic and Gulf coasts. The study also projects an even stronger population growth in the area in the future.

Encountering potential changes in frequency and severity of natural disasters, how to design mechanisms to transfer property exposures and ameliorate consequences of future catastrophes is a critical issue. Insurance is an efficient method to transfer risks. Insurers spread exposures by aggregating total risks into a relative larger risk pool to effectively reduce risk that can hardly be handled by a single homeowner. However, catastrophic risks pose special challenges for insurers since risks faced by each individual are not independent and the nature of catastrophic risks embarks on obstacles in accurate estimation, especially the extreme tail of the loss distribution. Insurers are usually forced to raise insurance premiums to reflect their difficulties in diversifying potential risks geographically or over time or voluntarily retreat from the insurance market in hazard-prone areas³. In addition to simply compensating damage, insurers can play a more proactive role in stimulating mitigation measures and enhancing economic resilience against natural disasters, with the assistance of public sectors. In this respect, insurers can regard these changes as a new business opportunity instead of a future threat and further develop innovative products for addressing potential changes.

In order to manage catastrophic risks, insurers typically purchase reinsurance for hedging against loss exposures caused by natural disasters. Reinsurers can spread a proportion of losses exceeding some level on a larger geographical area or on a greater risk pool which contains other low-correlated risks. Nevertheless, the prices of reinsurance for catastrophic risks are usually significantly high and the coverage is often limited, especially in the aftermath of great catastrophes. Alternatively, insurers can

³ In early 2009, State Farm, the largest property insurer in Florida, withdrew from the Florida property insurance market because regulators disapproved its proposal to increase premiums to reflect higher hurricane risks.

access additional capitals by trading Alternative Risk Transfer (ART) instruments, such as industry loss warranties (ILW), CAT bonds, CAT options, CAT futures, and other innovations in capital markets. Due to the relative huge capacity of financial markets and the presence of many active investors in these markets, securitization of catastrophic risks seems to be a promising solution. ART instruments allow insurers to transfer part of their exposure directly to capital markets as well as provide complementary sources of capital to absorb the losses resulting from mega catastrophes. For investors, the principal merit of ART instruments is that they constitute a different class of assets that can enhance returns while controlling the variance of a portfolio.

To turn risk management into value creation, the accelerating trend of large-scale catastrophes should incentivize new services, new types of protection, and innovative new financial solutions. The ideal financial solution is aimed at hedging against future catastrophic risks for insurers while sustaining availability and affordability of insurance. This dissertation proposes three different financial mechanisms in transferring and spreading catastrophic risks for insurers: Hybrid-trigger CAT bonds reduce basis risk compared with single-trigger CAT bonds; parimutuel insurance, albeit with some basis risk, eliminates the underlying risk assumed by insurers; longer term insurance, rather than annually renewal contracts, stimulates mitigation measures, thus effectively reduce risks and maintain insurability, even in the presence of potential climate change and related uncertainties.

In spite of recent evolutions and innovations of ART instruments, securitization still represents just a small share of overall capital in the global insurance market. Future development of insurance securitization markets depends on resolving current challenges.

First, enhance more accurate assessments of catastrophic risks. This can be done by building up a high-quality and comprehensive database on weather-related quantities and by constructing reliable catastrophic loss models across risk-prone areas. Second, create transparent and objective indices on catastrophic risks worldwide⁴. At present, hedge funds, catastrophe funds, and private equity funds are the major traders in insurance-linked security markets⁵. Clearly defined and regularly updated catastrophic-associated indices help to develop standardized products and derivatives, which further attract a broader range of investor base and enhance liquidity of the market. Third, customize insurance products that meet specific needs of investors. Tranching technique allows insurers to pool different risks and separate them into multiple layers based on their expected returns and offers investors a broad spectrum of tailored insurance options. Investor with various preferences on distinct risk-return profiles can purchase products with the appropriate layers of risks. In the long run, as long as these problems are ameliorated, these alternative financing devices will continue to bolster the supply of property insurance.

The reminder of this dissertation is structured as follows. Chapter 1 analyzes hybrid-trigger CAT bonds, a new CAT bond deal that can reduce basis risk and eliminate moral hazard simultaneously. The analytical and simulated evidence on the condition under which the hybrid trigger has lower basis risk are also provided. Chapter 2 examines

⁴ In 1992, the Chicago Board of Trade (CBOT) launched the first catastrophe futures and options after Hurricane Andrew. Due to thin trading volume, they failed to exist in the market. Initiate by the shortage of capital for insurance and reinsurance industry after 2005 hurricane season, the New York Mercantile Exchange (NYMEX) and the Chicago Mercantile Exchange (CME) start to trade U.S hurricane-related futures and options in 2007. NYMEX contracts are calculated based on PCS index while CME contracts are settled based on parametric indices in six U.S regions.

⁵ Based on Swiss Re (2007), these three types of investors represent 90% of the new issue volume of CAT bonds in 2006.

whether the parimutuel mechanism can hedge risk-averse people against catastrophic losses by constructing two optimal stake choice models, with stakes wagered by other individuals on the risk-prone areas exogenous and endogenous, respectively. The equivalent transaction costs of traditional insurance relative to HuRLOs are also derived to compare the effectiveness of hedging with traditional insurance and with parimutuels.

Chapter 3 considers climate change and the associated uncertainties into catastrophic risk models. Statistical properties of catastrophic losses in the presence of climate change, the benefit-cost analysis of mitigation measures under conditions of climate change on a longer time scale, and the impact of cost of capital and Bayesian-updated serial correlation on risk capital and insurance premiums for short and long term policies are further explored. This chapter intends to analyze the potential implications of climate change for catastrophic risks and to design appropriate risk transfer instruments to protect insurers against catastrophic losses and changes in risk estimates over time.

Chapter 1

Managing Catastrophic Losses for Insurers:

Hybrid-trigger CAT Bonds and Basis Risk Analysis

1.1 Introduction

1.1.1 Catastrophic Losses for Insurers

Catastrophic risks have been the focus of many property casualty insurers for the past two decades due to the dramatic surge in the frequency of catastrophe (CAT, thereafter) occurrence⁶. The representative events include Hurricane Andrew in 1992, Opal in 1995, Fran in 1996, Katrina, Rita, and Wilma in 2005, and the California earthquake in Northridge in 1994. The traditional hedging facilities that deal with the catastrophic risk applied by insurers are reinsurance contracts. Through reinsurance, insurers are compensated for partial or total losses incurred by catastrophic events. However, recent studies, including Cummins et al. (2002), Froot (1999, 2001), and Harrington and Niehaus (2003), have proposed that insufficient coverage is available in the reinsurance market for hedging against mega-CAT risks. Property liability insurance

⁶ As revealed in the Wharton Risk Center (2008), catastrophes have had a more devastating impact on insurers over the past 15 years than in the entire history of insurance. Before 1988, the worldwide insured losses from natural disasters were less than \$10 billion dollars. In contrast, after 1990, there was a radical increase in insured losses. 18 among the 20 most costly insured catastrophes worldwide from 1970 to 2006 occurred in the past 15 years.

companies have sought other funding sources to overcome this problem. Given that the US capital market is seventy-five times larger than the property insurance industry⁷, securitization of CAT risks offers a promising solution to the concern of insufficient capacity in the reinsurance industry to absorb the losses resulting from mega catastrophe in recent years. As a result, CAT-linked securities, such as Industry Loss Warranty (ILW), CAT options, and CAT bonds first appeared in the capital markets in the 1980s and 1990s. In addition to traditional reinsurance contracts, these securities together with Sidecars⁸ provide risk-transferring alternatives to ameliorate CAT shocks within the economy.

1.1.2 Nature of CAT Bonds

Among a variety of catastrophe-related securities, CAT bonds are the most popular. The CAT bond market, which first appeared in 1997, has become the major source of CAT risk transfer capacity in the capital market. According to newly published reports⁹, 2007 was by far the most active year in the history of the CAT bond market, with almost \$7 billion of issuance in 2007, another record established. In 2006, the issue volume, \$4.69 billion, broke the record of the issuing amount of the CAT bond market with a 136 percent increase over 2005's record performance of \$1.99 billion, and a 311 percent increase over the \$1.14 billion placed during 2004. The popularity of CAT bonds is not surprising since their payoff structure benefits both issuers (insurers) and buyers (investors). From the viewpoint of insurers, CAT bonds are appropriate hedging facilities

⁷ Based on Froot (1999)

⁸ Sidecars are financial structures which are created to allow investors to take on the risk and return of a group of insurance policies written by an insurer or reinsurer and earn the risk and return that arises from that business.

⁹ McGhee, C., Clarke, R., Collura, J. (2006), McGhee, C., Clarke, R., Fugit, J., Hathaway, J. (2007), and Lane and Bechwith (2007b)

because they provide capital release whenever insurers need a huge capital capacity to reimburse the coverage of their clients if CAT events occur. For investors, purchasing CAT bonds enhances the performance of their portfolios. Investors usually hold portfolios consisting chiefly of bonds and equities. A CAT risk can be regarded as close to a nonsystematic risk¹⁰ (or a zero-Beta risk), which is orthogonal to the systematic risk of the portfolio. Based on the standard financial economic theory, adding close to zero-Beta assets with a higher premium into a portfolio constructs a new portfolio with better performance compared to the original portfolio under the risk/return profile.

CAT bonds distinguish themselves from traditional bonds in the specific provision of debt-forgiveness. The debt-forgiveness provisions are triggered whenever the contracted CAT event occurs prior to the maturity of the bond. If debt-forgiveness provisions are triggered, CAT bond issuers are obligated to return only partial interest payments or principal thereafter until maturity. The trigger point can be determined by the actual losses suffered by the insurer (an indemnity-trigger) or a pre-specified index (a non-indemnity trigger), while the extent to which interest payments or principal are forgiven can be total, partial, or proportional to the actual losses. Moreover, within a single transaction, the trigger point can be activated by more than one type of indices (a hybrid trigger).

1.1.3 Advantages and Disadvantages of CAT Bonds

CAT bonds have been the most popular substitute of reinsurance contracts. Default risk, presented in reinsurance contracts due to the limited capital capacity of reinsurers, can be avoided by trading CAT bonds. Default risk plays a less role for both insurers and

¹⁰ Cummins and Geman (1995), Cox and Pedersen (2000), and Lee and Yu (2002)

investors in CAT bond transactions. Insurers face no default risk since they have the option, not the obligation, to reduce the interest payments or principal repayment whenever the debt-forgiveness provision is triggered. CAT bond investors bear less default risk because a special purpose vehicle (SPV), acting as an intermediary between the insurer and the investor, is regulated to deposit the principal into a trustee. The trustee guarantees the regular interest payments and the principal return at maturity, thus eliminating the possible default that arises from the insolvency of insurers. However, investors of CAT bonds still face the potential default risk when the trustee declares bankruptcy. For instance, after Lehman Brothers fell into bankruptcy in the recent financial turmoil¹¹, at least one CAT bond issue, Willow Re, has reportedly gone into default by missing interest payments to investors. Three other similar CAT bond deals managed by Lehman Brothers are approaching junk status and threaten to default. These CAT bond deals used a unit of Lehman Brothers as total return swap counterparty, contracted to ensure the collateral backing the bonds was sufficient to meet interest and principal payments, and to make up any shortfall. When Lehman Brothers collapsed and the collateral trust accounts were impaired by poor investments, investors were left with direct exposure to market losses on assets held as collateral.

The potential moral hazard problem and the basis risk of issuers are critical factors in determining the price of CAT bonds. As indicated in Doherty (1997), moral hazard behavior describes the insurers' incentive to exert less effort in loss control in order to gain from debt-forgiveness as the trigger event is approaching. Basis risk is a feature of non-indemnity-based CAT bonds. When the specified catastrophe occurs, the timing of

¹¹ Lehman Brothers filed for Chapter 11 bankruptcy protection on September 15, 2008.

reducing the interest payment and principal depends on specified indices instead of the insurers' actual losses. If the actual loss of the insurers is greater than the loss reflected on the index, the insurers' cash flows received from CAT bonds cannot cover their actual losses. As a result, insurers have to absorb the discrepancy of losses, leading to basis risk. The impact of basis risk on the CAT bond price is exclusively for non-indemnity triggers.

Non-indemnity trigger mechanisms¹² can be categorized into a parametric trigger, a PCS index trigger, and a modeled-based trigger. A parametric trigger evaluates losses based on certain defined physical parameters, like the wind speed or the magnitude of an earthquake (Richter magnitude scale). A PCS index trigger measures losses based on an aggregate industry index, estimated by Property Claim Services (PCS). A modeled-based trigger calculates the loss by inputting the physical parameters of actual catastrophes to a catastrophic risk modeling firm's¹³ database to estimate the loss exposure of its own or the whole industry. CAT bonds with an indemnity trigger have potential moral hazard problems while those with a non-indemnity trigger carry basis risks.

1.1.4 Hybrid-trigger CAT Bonds

Driven by the incentive to reduce basis risk and eliminate potential moral hazard behavior simultaneously, CAT bonds with a hybrid trigger, formed from the combination of two or more existing triggers, were introduced in the capital market in 2006. One

¹² McGhee, C., Clarke, R., Collura, J. (2006)

¹³ The scientific risk analysis and quantitative risk estimates of the catastrophe damage is developed by catastrophe-modeling firms. These firms build catastrophe models, which use meteorology, engineering, and insurance underwriting data to estimate damage in different areas. Input information is based on historical tropical storms, building construction, and the impact on various structures under different wind speeds. Today, the three leading proprietary catastrophe modeling firms are Risk Management Solutions (RMS), AIR Worldwide, and EQECAT.

example is the Calabash Ltd. Transaction¹⁴, which establishes the market share of losses by a model-based trigger, and the market share is then multiplied by the PCS-index to calculate the estimated loss. This version of CAT bonds combines a modeled-based and a PCS index trigger. In 2008, the Blue Coast, a unit of the insurer Allianz, issued a CAT bond with a similar trigger structure that allocates industry losses at the county level along the coastal areas of southeastern US. This \$120 million issue is the first to bring the losses induced by hurricanes down to the county level. A hybrid trigger which combines state-level PCS industry loss estimates and modeled county-level losses predicted by AIR Worldwide is used to define the trigger event. This specific structure enables insurers to manage basis risk at a more granular level while provides investors a transparent index, the PCS index, to gauge the catastrophic loss.

This study focuses on the comparative analysis of basis risk when using the hybrid trigger and other triggers and measures basis risk by taking the difference between the fair value of non-indemnity CAT bonds and that of indemnity-based CAT bonds to verify our analytical results. The principal merit of the hybrid-trigger CAT bond lies in that it possesses lower basis risk than the other non-indemnity CAT bonds. At the same time, it avoids moral hazard problem, the primary drawback of indemnity-based CAT bonds. Note that the meaning of ‘hybrid trigger’ is distinct from “hybrid parametric triggers”. The latter is now considered to be a parametric trigger that uses modeled parameters in the absence of the actual measured parameters of an extreme event.

¹⁴ This deal is sponsored by ACE American Insurance Co. It contains five classes of perils and two levels of series. Five classes are US wind, California earthquake, Central and North West earthquake, US earthquake, and all of the above. Series 1 is on an occurrence basis and Series 2 is on an aggregate basis. The loss measure is a modeled portfolio.

1.1.5 Basis Risk of Hybrid-trigger CAT Bonds

Basis risk measures the discrepancy between the actual loss of the insurer and the loss index specified to define trigger events. In the hybrid-trigger CAT bond deals we focus on in this paper, the loss index is determined by the modeled market share of losses and the PCS index. The hybrid trigger should naturally be compared with the PCS-index trigger and the model-based trigger in terms of basis risk since these two types of trigger involve the formation of ‘hybrid’ trigger. It has been claimed that lower basis risk is the primary advantage of hybrid-trigger CAT bonds over other non-indemnity CAT bonds¹⁵. However, no analysis has been conducted to illustrate this basic point. Intuitively, the hybrid trigger should produce less basis risk than the PCS index trigger. In the former deal, the market share of losses for the insurer reflects the most updated information, whereas in the latter deal, the market share of losses for the insurer is pre-determined at the inception of the contract. In contrast, the relative basis risk of the hybrid trigger and the model-based trigger is not straightforward. In the next section, basis risk will be defined and the conditions under which basis risk can be reduced for the hybrid trigger will be further explored.

1.2 Basis Risk Analysis for an Individual Insurer

Basis risk is defined as the absolute value of the difference between the losses used to define trigger events of CAT bonds and the actual losses for an insurer. Not only insurers but also investors of CAT bonds are concerned about basis risk because both

¹⁵ For instance, McGhee, Clarke, and Collura (2006) and Härdle and Cabrera (2009) stress this point to support the usage of hybrid-trigger CAT bonds.

counterparties could suffer from the potential losses of basis risk. Basically, it is a zero-sum game for insurers and investors in terms of basis risk in CAT bond deals. Insurers bear basis loss while investors attain basis gain, and vice versa. In this section, basis risk is measured from the view of the society. It arises as long as the actual losses mismatch the losses index pre-determined in the contract.

L_i is defined as the actual loss of insurer i during a year while \tilde{L}_i represents the loss of insurer i reflected in the loss index that is used to define trigger events during a year. L denotes the actual industry loss during a year, such as PCS index. \hat{L}_i and \hat{L} are the annual losses predicted by modeling firms for insurer i and for the insurance industry, respectively. a_i is the market share of insurer i . BR_j denotes basis risk in terms of different trigger types j , where $j=PCS, M$, or H . Based on these notations, basis risk= $\left|\tilde{L}_i - L_i\right|$ and basis risks for different trigger types are set as follows:

- Index trigger: $\tilde{L}_i = a_i \cdot L$, $BR_{PCS} = \left|a_i \cdot L - L_i\right|$ (1.1)

- Model-based trigger: $\tilde{L}_i = \hat{L}_i$, $BR_M = \left|\hat{L}_i - L_i\right|$ (1.2)

- Hybrid trigger: $\tilde{L}_i = \frac{\hat{L}_i}{\hat{L}} \cdot L$, $BR_H = \left|\frac{\hat{L}_i}{\hat{L}} \cdot L - L_i\right|$ (1.3)

Using hybrid-trigger CAT bonds may reduce basis risk and avoid moral hazard. Since the hybrid trigger is associated with two transparent and external triggers (the PCS-index trigger and the model-based trigger), moral hazard should not be a problem. Thus, this study emphasizes the analyses on basis risk. More specifically, the conditions under

which hybrid trigger generates less basis risk than the other two types of trigger will be derived.

1.2.1 Comparison of Basis Risk for the Hybrid Trigger and the PCS-index Trigger

Based on the definitions of basis risk, the hybrid trigger, and the PCS index trigger, the condition under which basis risk is less for a hybrid trigger than a PCS index trigger can be summarized as the following inequality:

$$BR_H < BR_{PCS} \Leftrightarrow \left| \frac{\hat{L}_i}{\hat{L}} - \frac{L_i}{L} \right| < \left| a - \frac{L_i}{L} \right| \quad (1.4)$$

As the modeled market share of losses borne by insurer i is closer to the actual market share of losses than the market share pre-determined in the contract, using the hybrid trigger leads to less basis risk compared to using the PCS index trigger. Consequently, the accuracy of modeling firms' prediction regarding the market share of losses of the insurer is critical in evaluating the relative basis risk of the hybrid trigger and the PCS-index Trigger. Since $BR_H < BR_{PCS}$ and $BR_H \geq BR_{PCS}$ are complementary events, the conditions that satisfy one is simply the complementary of the whole space of the other. Further analysis on the conditions by separate the space of $BR_H < BR_{PCS}$ into four disjoint situations is provided in section 1.5. Figure 1.1 depicts the areas of $BR_H < BR_{PCS}$ and $BR_H \geq BR_{PCS}$ using the actual market share of losses and the modeled market share of losses as x-axis and y-axis, respectively. The light areas stand for $BR_H \geq BR_{PCS}$ while the shaded areas represent $BR_H < BR_{PCS}$.

Take examples in Figure 1.1 to illustrate the analytical results. Suppose that the predetermined market share of losses for insurer i in the area is 20% according to the CAT bond contract, i.e., $a=0.2$ (the point A). As a catastrophe occurs and causes damage to properties with a proportion of only 15% compared to the industry loss, shown as the vertical line denoted with $\frac{L_i}{L} = 0.15$. This proportion is lower than the contracted market share of losses. The insurer receives more indemnity from the deal while compensate less claims to its clients, causing the insurer to attain basis gain. In this case, the modeled market share of loss should be within the range of 10% to 20% for guaranteeing that hybrid-trigger CAT bonds are more favorable to PCS-index CAT bonds in terms of basis risk. In mathematical terms, $BR_H < BR_{PCS} \Leftrightarrow 0.1 \leq \frac{\hat{L}_i}{\hat{L}} \leq 0.2$. In another case where a catastrophe strikes the same location but causes a more devastating damage for the insurer, which is equivalent to 40% of the total industry loss, i.e., $\frac{L_i}{L} = 0.4$, shown as the other vertical dashed line. The insurer suffers from basis loss at this time because the cash flow from the deal can not cover the claims from insurance clients. However, if the insurer uses the hybrid trigger to cover catastrophic loss exposures, basis risk can be reduced compared to using the PCS-index trigger even when the modeling firm predicts a wider range on the market share of losses, 20%-60%. This result implies that the more the actual market share of losses deviates from the contracted market share of losses, the hybrid trigger can still dominate the PCS-index trigger in terms of basis risk even if the modeling firm provides less precise prediction on the market share of losses.

In general, modeling firms' prediction on market share of losses for an individual insurer should be more accurate than the contracted market share of losses since modeling firms incorporate updated information and advanced approaches that combine various specialties, such as engineering, insurance underwriting, and meteorology or seismology. Based on this fact, along with the above analysis, trading based on the hybrid trigger essentially ameliorates basis risk rather than trading based on the PCS-index trigger.

1.2.2 Comparison of Basis Risk for the Hybrid Trigger and the Model-based Trigger

Using the aforementioned definitions on basis risk, the hybrid trigger, and the model-based trigger, the conditions under which basis risk is less for the hybrid trigger than a model-based trigger are illustrated as follows:

$$BR_H < BR_M \Leftrightarrow \left| \frac{\hat{L}_i}{\hat{L}} \cdot L - L_i \right| < \left| \hat{L}_i - L_i \right| \quad (1.5)$$

Detailed analyses on the conditions are also conducted in section 1.6. The results are summarized graphically in Figure 1.2. The shaded areas demonstrate $BR_H < BR_M$, and the light areas represent $BR_H \geq BR_M$. Note that the x-axis and y-axis are different from those in Figure 1.1. X-axis is the ratio of the modeled individual loss to the actual individual loss while the y-axis is the ratio of the modeled industry loss to the actual industry loss.

In contrast to the comparison of basis risk between using the hybrid trigger and using the PCS index trigger, the comparison of basis risk between using the hybrid trigger and using the model-based trigger in CAT bonds becomes more challenging in the sense that the analyses consider not only the relative market share of losses predicted by modeling

firms to the actual market share of losses, but also the precision of modeling firm's prediction on individual losses and on industry losses.

Figure 1.2 can be separated into four areas by $\frac{\hat{L}_i}{L_i} = 1$ and $\frac{\hat{L}}{L} = 1$, which imply the perfect prediction of the modeling firm on individual losses and industry losses, respectively. Area (1) ((2)) stands for the case that losses of the insurer are over-predicted (under-predicted) and losses of the industry that are under-predicted (over-predicted) by the modeling firm while area (3) ((4)) represents the case that both losses borne by the insurer and losses of the industry are under-predicted (over-predicted) by the modeling firm.

In area (1) and area (2), the hybrid trigger never has less basis risk than the model-based trigger due to the inconsistent prediction on individual losses and industry losses regarding the actual loss. The model-based trigger involves only the prediction of individual losses, but the hybrid trigger is influenced by the prediction of both individual and industry losses. The inconsistent predictions twist the estimate of loss, and thereby raise basis risk for the hybrid trigger. In area (3) ((4)), where both individual and industry losses are under-predicted (over-predicted), the hybrid trigger has less basis risk than the model-based trigger only if the difference of the actual market share of losses and the modeled market share of losses is small enough. Specifically,

$$\frac{\hat{L}_i}{\hat{L}} - \frac{L_i}{L} < \frac{L_i - \hat{L}_i}{L} \left(\text{or } \frac{L_i}{L} - \frac{\hat{L}_i}{\hat{L}} < \frac{\hat{L}_i - L_i}{L} \right).$$

'Hybrid-trigger favorable range' is defined as the length in the shaded areas in Figure 2 given a specific ratio of modeled loss to actual loss, either the individual loss or the

industry loss. For example, given that the ratio of the modeled individual loss to the actual individual loss is 0.5, the range is 1/3; whereas given that the same ratio is 1.5, the range becomes 2. This range reflects the positive signal in favor of using the hybrid trigger rather than using other triggers, in this case, the model-based trigger. With ‘hybrid-trigger favorable range’, two asymmetry phenomena that are found in the shaded areas in Figure 1.2 can be explained more explicitly.

The first phenomenon is associated with the asymmetry between the individual loss and the industry loss. More deviation from the perfect prediction on the individual loss expands the ‘hybrid-trigger favorable range’; however, more deviation from the perfect prediction on the industry loss reversely contracts the ‘hybrid-trigger favorable range’. This means that compared to model-based trigger, the hybrid trigger can tolerate more imprecision in estimating individual losses than in estimating industry losses. Moreover, it is more difficult to accurately estimate the individual loss than the industry loss due to the uncertainty of catastrophic losses across different risk-prone areas. The imprecision of predicted individual losses can be taken as the idiosyncratic risk while the imprecision of predicted industry losses can be regarded as the systematic risk. According to financial theory, the systematic risk is easier to be measured than the idiosyncratic risk. Thus, accurately estimating industry losses should generally be easier than accurately estimating individual losses for the modeling firm. The better accuracy of the industry loss along with the asymmetry between the individual loss and the industry loss strongly supports the argument that the hybrid trigger substantially reduces basis risk. This is a critical insight for insurers who seek CAT bond deals to cover catastrophic losses and are

concerned about basis risk because it provides theoretical evidence to promote the usage of the hybrid trigger.

The second asymmetry phenomenon is present by comparing the ‘hybrid-trigger favorable range’ in the over-predicted area and in the under-predicted areas. The range in the over-predicted area increases convexly while the range in the under-predicted area increases concavely if the individual loss parallel deviated from the perfect prediction¹⁶. Similarly, for the industry loss, the range in the over-predicted area declines convexly while the range in under-predicted area declines concavely. These findings indicate that compared to model-based trigger, the hybrid trigger can tolerate more imprecision for over-estimated losses than for under-estimated losses. Insurers who apply hybrid trigger should prefer a higher projection on the individual and industry by the modeling firm, even if it is not accurate, so as to better manage basis risk. This is not to say that the hybrid trigger performs worst than the model-based trigger if losses are under-estimated. The hybrid-trigger favorable area (the shaded area) is still larger in the under-predicted area compared with the hybrid trigger unfavorable area (the light area).

1.2.3 Basis Risk Analysis for Public Insurance/Reinsurance Program

Public sectors have been considered the last resort to absorbing catastrophic risks because they play an important role in guaranteeing insurability and should be responsible to provide disaster relief and assistance to victims after disasters. They also have quick access to cheap debt and the ability to diversify risks over the entire

¹⁶The hybrid-trigger favorable range is 1/3 if the ratio of the modeled individual loss to the actual individual loss is 0.5 while the range is 2 if the same ratio is 1.5. When the ratio of individual loss changes to 0.4, the range is 0.75. Nevertheless, as the same ratio parallel moves to 1.6, the range turn out to be 3. In the case of over-predicted area, the growth rate for a 0.1 level increase (from 1.5 to 1.6) in the ratio is 1 (from 2 to 3), whereas in the under-predicted area, the growth rate for the same level decrease (from 0.5 to 0.4) in the same ratio is only approximately 0.42 (from 0.33 to 0.75).

population and over future generations. Due to the nature of correlated risks and uncertainty in estimating impacts of radical natural disasters, public-private partnerships have been proposed to cover catastrophic risks¹⁷. Public insurance/reinsurance program is such an example. Representative ones include National Flood Insurance Program (NFIP)¹⁸ and Florida Hurricane Catastrophe Fund (FHCF)¹⁹.

Like private insurers, public insurance program can also seek other instruments, such as CAT bonds, to effectively transfer a proportion of the insured losses to capital markets and expand their financial capacity²⁰. Can the hybrid trigger still diminish basis risk compared with the other triggers, such as PCS-index trigger and the model-based trigger, for public insurance program?

We supposed that the public insurance program can be simplified as the monopolistic insurer, whose market share of losses is one. In this case, the hybrid trigger is simply equivalent to the PCS-index trigger. This can be verified in Figure 1.1. The whole space in Figure 1.1 turns out to be the northeast corner point, indicating no basis risk is present

¹⁷ Kunreuther (2006) and Kunreuther and Pauly (2006)

¹⁸ NFIP was established in 1968 for spreading flood risks nationally in US. It was originally designed as a voluntary partnership between federal government and communities: local governments enacted floodplain management regulations while property owners in participating communities were eligible for federal flood insurance. For setting premiums and designating flood risks through different flood zones, flood insurance rate map (FIRM) was created by NFIP. Furthermore, NFIP also run the Community Rating System (CRS) to promote mitigations by providing premium discounts for communities implementing mitigation measures. Florida represents the greatest proportion of NFIP coverage, with 40% of the policies in US. Refer to Chapter 4 of Kunreuther and Michel-Kerjan (2009) and Michel-Kerjan and Kousky (2009) for the history, statistics, and characteristics of demand on the program.

¹⁹ FHCF was established in 1992 in response to the threat of insurability after Hurricane Andrew. It provides catastrophe reinsurance to primary insurers underwriting property coverage in the state. This mandatory coverage program will reimburse a fixed percentage (45%, 75%, or 90%) of a participating insurer's losses from each covered event in excess of a per event retention and subject to a maximum aggregate limit for all events. The capacity of FHCF has been expanded to \$27.8 billion since January 2007. A detailed descriptions, payout calculation, and funding arrangement of FHCF are provided in Chapter 2 and Chapter 13 of Kunreuther and Michel-Kerjan (2009)

²⁰ In 2006, the Mexican government issued a parametric-trigger CAT bond to cover future earthquake losses up to three years to address escalated frequency of earthquakes in that areas, which could drain financial capacity of Mexico's Fund for Natural Disasters.

when using either the hybrid trigger or the PCS-index trigger. Nevertheless, the comparison of basis risk between the hybrid trigger and the model-based trigger in this extreme case suggests that the industry loss and the individual loss are the same. As can be seen in Figure 1.3, the whole space is condensed to the 45 degree line. Since the line lies within the ‘hybrid trigger favorable area’ (the shaded area), compared with model-based trigger, the monopolistic insurer who use the hybrid trigger bears less basis risk. In sum, for the public insurance program, which is assumed to be the monopolistic insurer, the hybrid trigger has the same basis risk with the PCS-index trigger but has less basis risk than the model-based trigger.

1.3 Basis Risk and Fair Values of CAT Bonds

After analyzing the basis risk of using a hybrid trigger versus using a PCS index trigger and using a model-based trigger under different scenarios, it is natural to investigate the relationship between basis risk and fair values of CAT bonds with a hybrid trigger and other trigger types. Since the indemnity-based CAT bond produces no basis risk for insurers, it can be regarded as the benchmark to compare the effect of basis risk. CAT bonds with a non-indemnity trigger could be priced lower or higher than indemnity-based CAT bonds depending on the setting of parameters. Hence, given the same payoff structure, the difference in the fair values of CAT bonds with a non-indemnity trigger and those with an indemnity trigger simply proxy the basis risk of the CAT bond with a non-indemnity trigger. The setting of loss dynamics, the contingent payoff, and fair values of CAT bonds are presented in the subsequent subsections.

1.3.1 Loss Model

Since default risks of CAT bonds are not the focus of this study, we postulate that asset dynamics of insurers do not affect the value of CAT bonds²¹. Moreover, we set the interest rate to be constant. For specifying CAT losses, consistent with previous studies²², we apply the Loss Distribution Approach (LDA), which assumes that loss frequency and loss severity are both identically independently distributed (i.i.d.) and mutually independent. Although dependence and correlations among aggregate loss models have been explored in recent research papers²³, especially in the operational risk area, we adopt the typical LDA approach, which assumes no correlations within and between loss frequency and loss severity for stressing the basis risk and avoiding unnecessary technical difficulties.

For indemnity-trigger, PCS index-trigger, model-based-trigger, and hybrid-trigger CAT bonds, the loss indices for each year are as follows:

$$L_{i,t}^I = L_i = \frac{L_i}{L} \cdot L = \frac{L_i}{L} \cdot \sum_{j=1}^{N_t} Y_j = \sum_{j=1}^{N_t} \alpha_i \cdot Y_j \quad (1.6)$$

$$L_{i,t}^{PCS} = a_i \cdot L = a_i \cdot \sum_{j=1}^{N_t} Y_j \quad (1.7)$$

$$L_{i,t}^M = \hat{L}_i = \frac{\hat{L}_i}{\hat{L}} \cdot \hat{L} = \frac{\hat{L}_i}{\hat{L}} \cdot \sum_{j=1}^{N_t} \hat{Y}_j = \sum_{j=1}^{N_t} \hat{\alpha}_i \cdot \hat{Y}_j \quad (1.8)$$

²¹ Lee and Yu (2002) analyze default risks on the price of CAT bonds and thus assume asset dynamic for insurers.

²² Baryshnikov, Mayo, and Taylor (2001), Lee and Yu (2002), Burnecki and Kukla (2003), and Härdle and Cabrera (2009)

²³ Recent papers include Frachot, Roncalli, and Salomon (2004), Chavez-Demoulin, Embrechts, Neslehova (2006), and Lindskog and McNeil (2001).

$$L_{i,t}^H = \frac{\hat{L}_i}{\hat{L}} \cdot L = \frac{\hat{L}_i}{\hat{L}} \cdot \sum_{j=1}^{N_t} Y_j = \sum_{j=1}^{N_t} \hat{\alpha}_i \cdot Y_j \quad (1.9)$$

α_i is the actual market share of losses for insurer i while $\hat{\alpha}_i$ denotes the market share of losses for insurer i estimated by the CAT modeling firm. N_t denotes the actual frequencies of hurricanes and is governed by Homogenous Poisson Process (HPP) with intensity λ . Y_j, \hat{Y}_j stand for the actual and estimated loss severities from hurricane j, respectively. Here we assume that the basis risk essentially comes from the bias in estimating loss severity rather than the loss frequency for the modeling firms.

1.3.2 Payoff of CAT Bonds

Contingent payoff of the coupon-paying CAT bond is supposed to be as follows.

$$Payoff = \begin{cases} X_t = c \cdot F, t = 1, \dots, T; X_T = F & \text{if } \tau > T, \text{ where } \tau = \min\{t : \tilde{L}_{i,t} > K\} \\ X_t = c \cdot F, t = 1, \dots, \tau - 1; X_\tau = (1 + c) \cdot f \cdot F & \text{if } \tau \leq T \end{cases} \quad (1.10)$$

K is the trigger point specified in the debt-forgiveness provisions of the CAT bond. $\tilde{L}_{i,t}$ denotes the aggregate loss index of insurer i at time t. If the trigger type is a PCS index, $\tilde{L}_i = a_i \cdot L$, where a_i is the contracted market share of losses for insurer i (typically set as the market share of insurer i in the insured area), and L is the PCS index (the actual industry losses); if the trigger type is model-based, $\tilde{L}_i = \hat{L}_i$, which is the estimated loss of insurer i forecasted by the CAT modeling firm in the insured area; if the trigger type is a hybrid-trigger, $\tilde{L}_i = \frac{\hat{L}_i}{\hat{L}} \cdot L$, where \hat{L} is the estimated aggregate loss of the insurance

industry provided by the CAT modeling firm in the insured area. The face value or the principal of the CAT bond is F ; c denotes the coupon rate; f is the fraction of coupon or principal payment if the forgiveness provision is triggered. The CAT bond matures at time T .

1.3.3 Fair Values of CAT Bonds

The fair value of CAT bonds can be derived by taking the discounted expectation of the contingent payoffs. In the case of zero-coupon CAT bonds, which pay no coupon before maturity, the implied coupon rates (or “default risks”) are reflected in the initial CAT bond prices. Higher “default risks” mean higher likelihood that the CAT bond is triggered, which drive down the price of the CAT bond. In other words, zero-coupon CAT bonds with lower prices are compensated with higher risk premiums. In contrast, since coupons are attached in the coupon-paying CAT bonds, prices are influenced by current level of interest rate and can not simply reflect potential “default risks” that are driven by trigger events.

Given the payoff specified above, based on Cox and Pedersen (2000), the risk-neutral price of CAT bonds can be presented as equation (1.11).

$$\begin{aligned}
 V &= E^Q \left(\sum_{k=1}^T \frac{1}{(1+r(0))(1+r(1)) \dots (1+r(k-1))} CF(k) \right) \\
 &= \sum_{k=1}^T B(k) \cdot E(\text{payoff}(k)) \\
 &= F \cdot \left[c \cdot \sum_{k=1}^T B(k) \cdot P(\tau > k) + B(T) \cdot P(\tau > T) + f \cdot (c+1) \cdot \sum_{k=1}^T B(k) \cdot P(\tau = k) \right]
 \end{aligned} \tag{1.11}$$

$B(k)$ is the zero-coupon bond with maturity k , where $k=1, \dots, T$. τ is the first time that the trigger event occurs, $\tau=1, \dots, T$. $P(\tau > k)$ measures the probability that the trigger event does not occur during the first k periods while $P(\tau = k)$ is the probability that the trigger event happens exactly at time k . In this study, the trigger event is defined as when the pre-determined loss proxy exceeds a threshold.

For specifying the trigger types, the prices of CAT bonds with different trigger types are shown as equation (1.12):

$$V^l = F \cdot \left[c \cdot \sum_{k=1}^T B(k) \cdot P(\tau^l > k) + B(T) \cdot P(\tau^l > T) + f \cdot (c+1) \cdot \sum_{k=1}^T B(k) \cdot P(\tau^l = k) \right] \quad (1.12)$$

, $l = I, PCS, M, \text{ or } H$

V^I, V^{PCS}, V^M and V^H ($\tau^I, \tau^{PCS}, \tau^M$ and τ^H) represent fair values (the first time that the trigger event occurs) of CAT bonds with an indemnity trigger, with a PCS index trigger, with a model-based trigger, and with a hybrid trigger, respectively. The probabilities exhibited in the pricing formula are associated with the loss indices in the manners of (1.13) and (1.14):

$$P(\tau^l > k) = P\left(\bigcap_{t=1}^k (L_{i,t}^l < \bar{L})\right), \text{ where } l = I, PCS, M, \text{ or } H \quad (1.13)$$

$$P(\tau^l = k) = P\left(\left(\bigcap_{t=1}^{k-1} (L_{i,t}^l < \bar{L})\right) \cap (L_{i,k}^l \geq \bar{L})\right) \quad (1.14)$$

It is worthwhile to note that CAT bonds have not been traded continuously in capital markets. Instead, they are traded by matching issuers and investors in the primary market.

The fair values presented here could be regarded as a reference in trading CAT bonds with different triggers despite the fact that CAT bond are empirically²⁴ overpriced.

1.3.4 Settings for Simulation

Monte Carlo Simulation can be conducted by setting a 1-in-100 year catastrophe, i.e. the annual probability of a catastrophe is 0.01. Thus, N_t is set to be a HHP with intensity rate $\lambda = 0.01$. Suppose a_i denotes the market share of insurance business for an insurer i . Once a catastrophe strikes a target area, it causes the insurer an insured damage that is equivalent to a proportion of α_i compared to the whole industry loss. Let actual share of the damage follows uniform distribution with mean 0.05, i.e., $\alpha_i \sim U(0.04, 0.06)$. The modeling firm's prediction on the loss share is uniform distributed with mean $\bar{\alpha}_i$, which is different from the actual market share, i.e. $\hat{\alpha}_i \sim U((0.8)\bar{\alpha}_i, (1.2)\bar{\alpha}_i)$. Moreover, the actual loss and the estimated loss predicted by the catastrophic risk modeling firm for the whole industry is correlated by setting (Y_j, \hat{Y}_j) to be bivariate lognormal distribution²⁵

with parameters $\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{pmatrix} \right)$, where $\mu_1=2$ and $\sigma=1$. The correlation coefficient of the actual loss and the modeled loss, ρ , varies from 0.1 to 1.0 with an increment of 0.1.

Suppose the face value of the CAT bond is \$100, with coupon rate 0.05, and will mature

²⁴ Lane and Maheul (2008) empirically examine the extent to which the catastrophic risk is overpriced based on 247 insurance-link securities issued after 1997. They found that the capital market generally estimates catastrophic risk premium to be 2 to 3 times the expected loss with and without conditioning on the market cycle.

²⁵ Bivariate g-and-h distribution could be another potential candidate to model the severity of losses. G-and-h family of distributions, introduced by Tukey (1977), has been extensively used in finance field to model return on equity, including Badrinath and Chattejee (1988), Mills (1995), and Badrinath and Chatterjee (1991), interest rates and interest rate options, like Dutta and Babbel (2002, 2005), and operational risks, such as Dutta and Perry (2007). The attractiveness of this distribution is that it can span over almost all areas of skewness and kurtosis dimensions. In other words, it can fit almost all parametric distributions up to fourth moments. Loss severities show heavy-tailed shapes empirically. Thereby, g-and-h distribution seems to be the ideal model for aggregate losses.

in 5 years. The forgiven provision of the CAT bond will be triggered if the loss is greater than \bar{L} . \bar{L} takes values from 0.2 to 2.0 with an increment of 0.2. The trigger event causes the payments of coupon and principal being reduced to 20% of the original level.

1.3.5 Simulation Results

1.3.5.1 PCS-index CAT Bond and Hybrid-trigger CAT Bond

According to the analysis in section 1.2.1, PCS-index CAT bonds possess less basis risk than hybrid-trigger CAT bonds if the contracted share of loss is accidentally more close to actual share of loss for the insurer than the market share of loss projected by the modeling firm. For conducting the simulation, two cases are considered: (1) the discrepancy between the contracted share of loss and the actual share of loss is greater than the discrepancy between the modeled share of loss and the actual share of loss. Specifically, $a_i = 0.02, \bar{\alpha}_i = 0.07$ (2) the discrepancy between the contracted share of loss and the actual share of loss is smaller than the discrepancy between the modeled share of loss and the actual share of loss. Specifically, $a_i = 0.07, \bar{\alpha}_i = 0.08$. To simplify the comparison, the actual industry loss is assumed to have the same mean with the modeled predicted industry loss, i.e., $\mu_1 = \mu_2 = 2$.

Table 1.1(1.2) exhibits the simulation results of case (1) ((2)). Panel A, B, and C shows the fair values of indemnity-based, PCS-index, and hybrid-trigger CAT bonds, respectively while Panel D (E) presents the price differential between the indemnity-based CAT bond and the PCS-index (hybrid-trigger) CAT bond. The quantities in Panel D and Panel E are the most critical results in these tables. The comparison of the absolute values of the quantities in Panel D and in Panel E reflects the relative basis risk using

PCS-index CAT bonds and hybrid-trigger CAT bonds. The absolute values of the numbers in Panel D of Table 1.1(1.2) are greater (less) than those in Panel E, meaning that the insurers who apply PCS-index CAT bonds to transfer their exposure to catastrophic risks bear more (less) basis risk than if they issue hybrid-trigger CAT bonds. Generally, the projection of the modeling firm should be more precise than the contracted market share that is determined at the initiation of CAT bond deals because modeling firms dynamically update information on catastrophic risks. The situation of case (2) may occur simply by luck or due to the mismatch of actual loss and the modeled loss. These results are essentially consistent with the analytical results in section 1.2.1. A comparison of basis risk resulted from using hybrid-trigger CAT bonds and PCS-index CAT bonds basically depends on the market share of loss projected by catastrophic risk modeling firms and the market share of loss pre-determined in the contract. If the actual market share of loss turns out to be closer to the projected market share than the pre-determined market share, hybrid-trigger CAT bonds is favorable over PCS-index CAT bonds, and vice versa. Figure 1.4 (1.6) shows the difference of prices between indemnity-base CAT bonds and PCS-index CAT bonds while Figure 1.5 (1.7) demonstrates the difference of prices between indemnity-based CAT bonds and hybrid-trigger CAT bonds in case (1) (case(2)).

1.3.5.2 Model-based CAT Bond and Hybrid-trigger CAT Bond

In order to verify the analytical results derived in section 1.2.2, the setting of μ_2 and $\bar{\alpha}_i$ are based on the four cases: (3) The pair $(\mu_2 = 1.8, \bar{\alpha}_i = 0.08)$ corresponds to $E(\hat{L}_i) > E(L_i), E(\hat{L}) < E(L)$. (4) The pair $(\mu_2 = 2.2, \bar{\alpha}_i = 0.02)$ corresponds

to $E(\hat{L}_i) < E(L_i), E(\hat{L}) > E(L)$. (5) The pair $(\mu_2 = 1.8, \bar{\alpha}_i = 0.02)$ corresponds to $E(\hat{L}_i) < E(L_i), E(\hat{L}) < E(L)$. (6) The pair $(\mu_2 = 2.2, \bar{\alpha}_i = 0.08)$ corresponds to $E(\hat{L}_i) > E(L_i), E(\hat{L}) > E(L)$.

The prices of CAT bonds with different triggers are shown in Figure 1.8-1.11. Since the basic patterns are similar for the four cases, only the case (3) is illustrated.

Table 1.3-1.6 report parts of the simulation results in four cases. Fair values of indemnity-based, model-based, and hybrid-trigger CAT bonds are presented in Panel A, B, and C, respectively. Panel D (E) provides the difference of prices between the indemnity-based CAT bond and the model-based (hybrid-trigger) CAT bond. A higher value on the absolute value of the quantities in Panel D compared to the absolute value of the corresponding quantities in Panel E indicates that model-based CAT bonds have higher basis risk than hybrid-trigger CAT bonds. As anticipated, hybrid-trigger CAT bonds have more basis risk because the predictions on individual losses and industry losses of catastrophic risk modeling firm show inconsistency (overestimate one while underestimate the other) for most simulation samples in case (3) and (4); however, holding hybrid-trigger CAT bonds is more likely to diminish basis risk in case (5) and (6). In addition, the reduction of basis risk for hybrid-trigger CAT bonds is more pronounced in case (6) than in case (5), concordant with the second asymmetry phenomenon in section 1.2.2. The ability of diminishing basis risk for hybrid-trigger CAT bonds is stronger if losses are over-predicted than if losses are under-predicted. Take the case where the correlation coefficient is 0.5 and the threshold is 1 as an example. In Table 1.5, the measure of basis risk of a hybrid-trigger CAT bond is 0.0835,

lower than that of a model-based CAT bond, which takes value 0.091. In contrast, in Table 1.6, the measure of basis risk of a hybrid-trigger CAT is 0.0862 while that of a model-based CAT bond is 0.1394. Thus, a much greater reduction in basis risk can be observed if most simulation loss samples are overestimated (case (6)) than underestimated (case (5)).

The discrepancies of the prices of model-based (hybrid-trigger) CAT bonds and indemnity-based CAT bonds are important measures of basis risk, so they are exhibited separately for four cases in Figure 1.12-1.19. A crucial finding through these figures is that prices of CAT bonds and basis risk are more responsive to the threshold than the correlation. Basis risk can be greatly reduced in the presence of a higher threshold, meaning that insurers who hedge catastrophic risk by means of the model-based or hybrid-trigger CAT bonds can substantially reduce basis risk by raising the threshold at the inception of the deal rather than focusing on the precision of the projected industry loss provided by the modeling firms. Another finding can be observed by making two pairs of comparisons: the comparison between case (3) and case (5) and the comparison between case (4) and case (6). Basis risk is higher in case (4) (case (5)) than that in case (6) (case (3)) if the threshold of CAT bonds is low. This implies that a lower threshold tends to raise basis risk when the modeling firm under-predicts the market share of individual loss compared to when it over-predicts the market share of individual loss. When the threshold is set to be low, insurers will bear more basis risk if the modeling firm projects market share of loss that is lower than actual share of loss. These implications are important for insurers to manage basis risk when they use CAT bonds as a hedging instrument to transfer catastrophic risks to capital markets. It is a best situation

for insurers if they can issue CAT bonds with higher thresholds; however, if markets prefer CAT bonds with lower thresholds, insurers who issue those bonds have to alert the prediction of the individual loss. In terms of basis risk, it is more detrimental for insurers as the catastrophic risk modeling firm tends to under-predict rather than over-predict their shares of losses within the industry.

1.4 Conclusions and Discussion

Moral hazard and basis risk are two major problems that impede the development of CAT bond markets. Investors are concerned about moral hazard when they trade indemnity-based CAT bonds while insurers may face basis risk when they issue CAT bonds with triggers determined by an external index. Both create dead-weight-loss to the social welfare²⁶. A product that can ameliorate these two problems at the same time will create value. Because hybrid trigger is constructed by two external indices, investors would unconcern about moral hazard if it is used to determine the loss in CAT bond deals. Nevertheless, insurers may hesitate to issue this type of CAT bonds due to the inherent basis risk. Less basis risk will reduce the hedging cost to insurers. Insurers can thus focus on their core businesses without paying too much attention on hedging basis risk. But it is not clear when the hybrid trigger will reduce basis risk compared with other

²⁶ More specifically, for moral hazard, if the trigger can be controlled by insurers, as in the case of indemnity-based CAT bonds, when the loss is less than but close to the threshold, insurers will take advantage of investors by inputting less effort in loss control in order to trigger the CAT bond. Investors know that insurers will behave in this way so would purchase less CAT bonds. For basis risk, investors may want insurers to use CAT bonds as a hedge because CAT bonds not only may provide a higher return but also have low correlations with the existing asset classes. However, insurers may be less willing to issue CAT bonds due to their concern with basis risk. Investors will be worse off than if insurers issues CAT bonds as a new asset class. In both cases, CAT bond deals will transfer less risk than the optimal level when the market is efficient (no moral hazard and no basis risk).

triggers. (e.g. PCS index and modeled-based triggers.) The hybrid trigger is determined by the market share of loss predicted by the catastrophic risk modeling firms and the PCS index. The principal advantage of hybrid-trigger CAT bonds is that it is claimed to avoid moral hazard and diminish basis risk simultaneously, but no analysis has been conducted with respect to its basis risk. This is the first paper that provides analytical results along with simulation evidence on the conditions that the hybrid trigger effectively reduces basis risk for insurers. Based on my analyses, the accuracy of the modeled loss relative to the actual loss at the industry level and at the individual level is critical in determining the relative performance of hybrid trigger in terms of basis risk.

Two types of triggers that involve the formation of hybrid trigger are eligible for the comparison: the PCS-index trigger and the model-based trigger. The analytical results shows that the hybrid trigger has lower basis risk than PCS-index trigger as long as the market share of loss predicted by the modeling firm is more accurate than the pre-determined market share of loss. However, the hybrid trigger never produces less basis risk than the model-based trigger if the industry loss and individual loss exhibit inconsistency for the actual loss and the predicted loss by the modeling firm (overestimate one while underestimate the other). In the case of consistency between the actual loss and the modeled loss, when the projection on the industry loss is more precise than the projection on the individual loss, the hybrid trigger could substantially reduce basis risk compared with the model-based trigger. Moreover, insurers should prefer overestimated losses to underestimated losses.

Due to the absence of basis risk for the indemnity-trigger CAT bonds, they are taken as the benchmark to gauge the basis risk of CAT bonds with different triggering structures.

The proxy of basis risk for the non-indemnity CAT bonds is simply the difference between fair values of non-indemnity CAT bonds and indemnity-based CAT bonds.

The simulation results support our analyses on the basis risk of the hybrid trigger in comparisons with both the PCS-index trigger and the model-based trigger. These results are robust with respect to different thresholds and correlations between the actual loss and modeled loss. Additionally, the proxy of basis risk is more sensitive to the threshold than the correlation. This suggests that the insurers who use hybrid-trigger CAT bonds should pay attention to the level of threshold rather than the accuracy of the modeled industry loss. In the case of CAT bond deals with a lower threshold, insurers will encounter more basis risk if modeled losses are under-estimated than if they are over-estimated.

In this study, we analyze the basis risk of different triggers in terms of the discrepancy between the realized loss and the losses implied by different triggers. However, the realized losses cannot be observed until the loss. Prior to the losses being realized, we can only estimate the modeled loss and the actual loss by loss distributions. Like in simulations, we can only specify the expected modeled losses being greater than the expected actual losses, but cannot guarantee that all modeled loss samples exceed actual loss samples. Thus, the simulations only provide general results regarding the analyses on basis risk. In order to explore the statistical properties that will a trigger lower basis risk, we need to assign loss distributions to the modeled loss and the actual loss and define basis risk as the expected value of the discrepancy between these two losses. We can then start to analyze practical problems such as the impact of the correlation of actual losses and modeled losses on the basis risk with different triggers, the impact of volatilities of

industry losses and individual losses on the basis risk with different triggers, and the optimal conditions under which a specific trigger will have less basis risk.

Even with this drawback, my analysis provides valuable information for financial institutions who hedge catastrophic risk through CAT bonds. Under some circumstances, hybrid-trigger CAT bonds provide a better hedge choice among the existing CAT bonds against basis risk. First, if an insurer is considering hedge with PCS-index CAT bonds and believes that the market share of losses predicted by the modeling firms is more likely close to the actual market share of losses than what the contract indicates, hybrid-trigger CAT bonds will be a better choice. Second, if an insurer wants to hedge with modeled-based CAT bonds and believes that the modeling firm will estimate the industry loss very well and will overestimate (or underestimate) the industry loss and the individual loss at the same time, he will be better off to choose hybrid-trigger CAT bonds. But, if the insurer believes that the modeling firm will overestimate (underestimate) industry losses but underestimate (overestimate) individual losses, he should stick to the modeled-based CAT bonds.

The major results and findings in this study are useful for insurers and financial institutions involving the transactions of CAT bonds. Especially, they provide insights to insurers who would like to proactively manage basis risk when issuing CAT bonds with hybrid trigger or other triggering structures. However, the simulation results may not robust with respect to different settings of loss distributions. Further analysis on the basis risk using different loss distributions would be helpful to clarify that under which statistical property will the hybrid trigger has lower basis risk.

Appendix

1.5 Analysis on the Conditions where the Hybrid Trigger Has Less Basis

Risk than the PCS Index

$$BR_H < BR_{PCS} \Leftrightarrow \left| \frac{\hat{L}_i}{\hat{L}} - \frac{L_i}{L} \right| < \left| a - \frac{L_i}{L} \right|$$

$$\text{Let } \frac{\hat{L}_i}{\hat{L}} = Y, \frac{L_i}{L} = X$$

The range that satisfies the objective can be transformed into

$$|Y - X| < |a - X|, \quad 0 < X, Y < 1$$

The whole space is divided into four disjoint situations in order to show explicitly the areas that satisfy the conditions under which basis risk is lower for the hybrid trigger than for the PCS-index trigger.

$$(1) Y - X > 0 \quad \& \quad a - X > 0$$

$$Y - X < a - X \Rightarrow Y < a$$

$$(2) Y - X > 0 \quad \& \quad a - X < 0$$

$$Y - X < X - a \Rightarrow Y < 2X - a$$

$$(3) Y - X < 0 \quad \& \quad a - X > 0$$

$$X - Y < a - X \Rightarrow Y > 2X - a$$

$$(4) Y - X < 0 \quad \& \quad a - X < 0$$

$$X - Y < X - a \Rightarrow Y > a$$

The shaded areas Figure 1.20 depict the conditions under which basis risk is lower for the hybrid trigger compared to the PCS-index trigger in the case where $a=0.2$. Areas (1)-(4) are corresponding to the four areas we described above. These areas are divided by two intersected lines, $\frac{\hat{L}_i}{\hat{L}} = \frac{L_i}{L}$ and $\frac{L_i}{L} = a = 0.2$.

1.6 Analysis on the Conditions where the Hybrid Trigger Has Less Basis Risk than the Model-based Trigger

$$BR_H < BR_M \Leftrightarrow \left| \frac{\hat{L}_i}{\hat{L}} \cdot L - L_i \right| < \left| \hat{L}_i - L_i \right| \Leftrightarrow \left| \frac{\hat{L}_i}{L_i} \cdot \frac{L}{\hat{L}} - 1 \right| < \left| \frac{\hat{L}_i}{L_i} - 1 \right|$$

$$\text{Let } \frac{\hat{L}}{L} = Y, \frac{\hat{L}_i}{L_i} = X$$

The range that satisfies the above inequality can be transformed into

$$\left| \frac{X}{Y} - 1 \right| < |X - 1|, \text{ where } X > 0, Y > 0$$

Similarly, the whole space is divided into four disjoint situations in order to show explicitly the areas that satisfy the conditions under which basis risk is lower for the hybrid trigger than for the model-based trigger.

$$(1) \frac{X}{Y} - 1 > 0 \ \& \ X - 1 > 0$$

$$\frac{X}{Y} - 1 < X - 1 \Rightarrow 1 < Y$$

$$(2) \frac{X}{Y} - 1 > 0 \quad \& \quad X - 1 < 0$$

$$\frac{X}{Y} - 1 < 1 - X \Rightarrow X \left(\frac{1}{Y} + 1 \right) < 2 \Rightarrow \frac{2 - X}{X} < Y$$

$$(3) \frac{X}{Y} - 1 < 0 \quad \& \quad X - 1 > 0$$

$$1 - \frac{X}{Y} < X - 1 \Rightarrow 2 < X \left(\frac{1}{Y} + 1 \right) \Rightarrow Y < \frac{2 - X}{X}$$

$$(4) \frac{X}{Y} - 1 < 0 \quad \& \quad X - 1 < 0$$

$$1 - \frac{X}{Y} < 1 - X \Rightarrow Y < 1$$

Figure 1.21 shows the conditions under which basis risk is lower for hybrid-trigger CAT bonds compared to mode CAT bonds in the shaded areas. Areas (1)-(4), as defined

above, are divided by the two lines: $\frac{\hat{L}}{L} = \frac{\hat{L}_i}{L_i}$ and $\frac{\hat{L}_i}{L_i} = 1$. Note that the areas (1)-(4)

defined here are not consistent with the four areas in the context and in Figure 1.2 and Figure 1.3. The aim for defining the areas here is simply for calculation convenience.

Chapter 2

Parimutuel Insurance for Hedging against Catastrophic Risk

2.1 Introduction

People and firms can insure against catastrophes such as hurricanes in the United States. However, for various reasons, this market does have limited capacity. This limitation is due to many factors; the high geographic concentration of insured risk; state regulatory strategies that constrain prices, and various types of transaction costs. To supplement the insurance market, a very different type of financial instrument has recently emerged for taking positions on hurricane landfalls; this is a parimutuel market place and the new instruments are known as HuRLOs (Hurricane Risk Landfall Options). While this structure resembles parimutuel horse race betting, the reasoning behind the opening of the HuRLO market was to provide further hedging capacity to those exposed to risk.

The new HuRLO instruments can be classified as an Insurance Linked Securities (ILS's), thus supplementing the existing instruments such as industry loss warranties, catastrophe options, and catastrophe bonds. These existing ILS's are targeted at insurance companies and large investors. Insurers use ILS's to provide hedge capacity (supplementary to reinsurance), and for portfolio diversification. And investors, such as

hedge funds, are now treating natural catastrophe risk as a new asset class that has a potential for high returns and which has low correlations with existing asset classes. What is very different about the HuRLOs, is that while they can be used by similar stakeholders (insurance companies and large investors), they are now being made available to smaller players such as individual householders and small businesses. The reason for targeting these smaller players is to supplement existing insurance protection which may be limited. Such limitations can occur in various ways such as large deductibles, the effects of regulatory cross subsidies built into premiums, or the failure of business interruption insurance to make payments unless the policyholder's own property is damaged.

On its face, a parimutuel does not seem an ideal instrument for hedging since it has inbuilt basis risk. Payments do not depend on whether the individual suffers a financial loss, only on a hurricane landfall occurs in the location chosen. However, we will show that, for quite reasonable parameters, these parimutuel "bets" might sometimes offer higher expected utility to risk-averse householders or small business owners, than traditional insurance. The issue boils down to a trade-off between basis risk and transaction costs. We will show that, for individuals with rather representative levels of risk aversion, HuRLOs might offer higher expected utility than traditional insurance, when the insurance transaction costs exceed 20-25%.

The parimutuel mechanism was invented by Pierre Oller in 1865 as a betting system which guarantees a fixed profit for bookmakers. Nowadays, the parimutuel wagering system is adopted in at least 37 nations worldwide with trading volume up to \$100 billion

per year. Since 2002, various investment banks²⁷ have also employed the parimutuel mechanism in trading options on economic statistics, such as the US non-farm payroll report, Euro-zone harmonized inflation, and speed at which the Fannie Mae mortgage pool receives prepayment. Most recently, on October 7th, 2008, an innovative commodity option that applies the parimutuel was launched by Weather Risk Solutions, LLC (WRS); it is called Hurricane Risk Landfall Options (HuRLOs)²⁸. The HuRLO market provides individuals the opportunity to hedge against, or speculate on, the risk that a specific region in the Gulf of Mexico and on the East Coast of the U.S. between the Mexican and Canadian borders will either be first hit by a hurricane or no hurricane will occur during a year. The HuRLOs market opened in early January 2009, continues into the hurricane season, and closes when a hurricane makes landfall or no hurricane makes landfall before the middle of December among the covered regions²⁹. As a hurricane approaches so close to a specific region that the National Hurricane Center (NHC) issues a hurricane warning, trading will be suspended until the hurricane makes landfall or the imminent threat of the hurricane landfall abates. The risk pools are separated for different series of a hurricane's landfall. For example, participants in Series 1 (2) HuRLOs involve the risk of the landing of the first (second) hurricane.

²⁷ Deutsche Bank and Goldman Sachs introduced the first Parimutuel Derivative Call Auctions of options on economic data releases, including employment, industrial production, economic growth, inflection, etc.

²⁸ In 2008, HuRLOs trade via the WRS electronic trading platform on the Chicago Mercantile Exchange Alternative Marketplace, Inc.'s (CME AM) exempt board of trade (EXBOT) and are cleared by the CME. Their prices are determined first based on the historical hurricane risk database and then interact dynamically by trading decisions made by all participants via a mathematical adaptive control algorithm that adjusts in a way that makes the selected outcome in the last trade more expensive and other outcomes less expensive. Detailed procedures of the algorithm are presented in Meyer et al. (2008).

²⁹ In 2008, HuRLOs market began in the latter part of the hurricane season.

In order to facilitating the transaction of HuRLOs, WRS also provides market-based and forecast-based landfall probabilities for each covered region on its website³⁰. Market-based probabilities that reflect the price for a single option will change with each purchase in a particular risk pool. Forecast-based probabilities are estimated based on historical data and current weather conditions. When no current hurricane is identified, these estimates simply reflect historical information; otherwise, these probabilities are based on current NHC forecast data.

Both hedgers who possess assets in the hazard-prone areas, and speculators who would like to bet on which covered area will first be hit by a hurricane, may be interested in trading HuRLOs. The participation of speculators enhances the market liquidity and expands risk pools. While, in this paper, we only address hedgers, a paper by Meyer et al. (2008) examines the parimutuel market with only speculators. They test empirically the HuRLO market in a controlled laboratory experiment, where participants are allowed to buy HuRLOs to maximize their profits in a simulated hurricane season. With limited trading experience, market prices converge quickly to objective probabilities of hurricane landfalls, suggesting that these trades act efficiently. Most potential investment anomalies³¹ are not empirically verified. However, the boomerang bias, where the “no

³⁰ <http://www.weatherrisksolutions.com/>

³¹ Investment anomalies examined include procrastination biases, distorted beliefs about probabilities, speculative bubbles, and false-alarm biases. Procrastination represents investors' preference for purchasing HuRLOs until a storm is formed and threatens a specific area. This bias could decrease the size of a mutualized risk pool and increase the uncertainty to returns. Although the HuRLO market provides objective landfall probability for each area, a distorted belief about probabilities could arise due to the perception bias of investors. Speculators believe that the higher price of HuRLOs in a specific area relative to its objective probability could reflect the private information held by other investors, further raising the option price and creating bubbles to the price. False-alarm bias describes the situation where no landfall in a specific area that was thought to be likely would discourage later investment in that area.

landfall” option is overpriced immediately after a hurricane passes through a specific region, is observed according to the experimental results.

The paper proceeds as follows. Section 2.2 presents the introduction of the parimutuel mechanism. Section 2.3 compares parimutuels with traditional insurance. In Section 2.4, given that stakes from others are exogenous, we formulate a model for a risk-averse individual who would like to hedge against potential hurricane risk by parimutuels and analyze his optimal stake choice. Section 2.5 considers the case in which a risk-averse representative hedger uses parimutuels to insure his potential loss from a hurricane. The equilibrium of parimutuel stake and the comparative static analysis are provided. Section 2.6 summarizes the pros and cons of parimutuel insurance. Section 2.7 discusses the role of speculators. Section 2.8 estimates the equivalent transaction costs of traditional insurance relative to HuRLOs. Section 2.9 concludes.

2.2 The Parimutuel Mechanism

The parimutuel mechanism is a betting system where bettors wager on one of the exhaustive and mutually exclusive possible outcomes before the race begins. As the outcome of the race is realized, the total stakes of bettors are distributed among the bettors who wager on the winner of the race, in proportion to the initial stakes. Bettors who wager on other horses lose their stakes. In practice, bookmakers usually deduct a certain percentage of the total amount of stakes as taxes and transaction fees, called the track take. Suppose there are S exhaustive and mutually exclusive outcomes with m_s wagers on outcome s , where $s=1, 2, \dots, S$. As outcome k turns out to be the winner, each

bettor who wagers on outcome k receives $\sum_{s=1}^S m_s / m_k$ for each dollar he wagered; while bettors who wager on outcomes other than k receive nothing. To put it differently, the winning bet is refunded with his original stake and an extra return stemming from equally sharing the stakes of the losers. This extra return, $\sum_{s \neq k} m_s / m_k$, also known as odds, shows the net return that will be paid out to the bettor, should he win, relative to his stake.

A framework of the mathematical principles of parimutuel pricing and their implications is presented in Baron and Lange (2007). These include the arbitrage-free and the self-hedging principles. The arbitrage-free principle describes two conditions under which parimutuel participants can not profit without taking any risk: the price of each state is positive, and the prices of all states are summed to be one. The former condition precludes the possibility of making profit by wagering on any single state, whereas the latter rules out the possibility of making profit by wagering on all states simultaneously. The self-hedging principle states that all payouts of the parimutuel game come from the initial wagers. Without a transaction fee and tax, the parimutuel mechanism is a zero-sum game for participants. This feature appeals to market makers, who bear no risk. By these two principles, the price of the state s claim (or the implied probability of state s) is the proportion of stakes wagered on the state s relative to the stakes wagered across all states. Given that the total stakes across all states are the same, the relative stakes wagered on these states reflect the relative prices of the state claims. More stakes wagered on one state relative to another represent a higher price of the state relative to the other. This property is referred to as the relative-demand pricing since the stakes wagered on a state claim just reflect its relative demand.

Because horse racing was the earliest and the most popular wagering system to adopt the parimutuel mechanism, we demonstrate the general procedures on horse racing. The wagering period begins 20 to 30 minutes before the upcoming race starts and closes as soon as the race starts. During that period, bettors submit their target horse and wagers to bet. Payoff to the winning bet is announced after the race is finished and the official results are finalized. The odds, standing for the bettor's net returns on the specific horse, are inversely related to the state prices (or implied probability of that state). If all other conditions remain constant, the more wagers there are on one horse, the lower the odds are against that horse, but the higher the odds are against the other horses. Typically, the odds are exhibited on the track tote board in the form of odds to 1. During the wagering period, odds change over time with incoming wagers on different horses. Only the final odds, revealed at the end of the wagering period, reflect the net returns of the bets on horses. In this sense, bettors do not exactly know the claim price (or payoff) when they place their bet, and only find out when the wagering period closes.

In addition to wagering on a horse to win, bettors can wager on a horse to place or to show, where the bettors win the game if their target horse ends up in the top two or three positions, respectively. In practice, the win, place, and show wagers are pooled separately. This separate pooling creates arbitrage opportunities and reduces liquidity. As a result, extensive studies have been devoted to searching for systematic strategies to beat the parimutuel game³².

³² With a high track take, approximately 15-25% of aggregate wagers, making a profit based on the odds and information inside the wagering pools seems to be a daunting task. However, Hausch, Ziemba, and Rubinstein (1981), Hausch and Ziemba (1985), and Ziemba and Hausch (1987) discovered profitable strategies, widely known as the "Dr. Z system", based on their finding that horses likely to win are underbet in place and show wagers. Asch, Malkiel, and Quandt (1984, 1986) and Asch and Quandt (1986) developed

Like capital markets, wagering markets contain a large number of participants and a variety of information sources. In addition, they have a well-defined termination point where eventual payoffs are determined. This feature offers economists an ideal environment to examine market efficiency and to observe human behavior under conditions of uncertainty. In contrast, security prices in capital markets change continuously and are affected by various uncertain factors, such as the future cash flow of firms and relative demand/supply structure in the markets.

Several market anomalies³³ have been observed in examining the market efficiency hypothesis of wagering markets. The most common one is the favorite-longshot bias, in Griffith (1949) and subsequent papers³⁴. This bias is characterized by favorites (horses with short odds to win) that win more frequently than the odds predict and longshots (horses with long odds to win) that win less frequently than the odds predicts.

Consistent with the convex empirical utility function constructed by Weitzman (1965) and Ali (1977), the favorite-longshot bias reflects the risk-seeking behavior of participants in wagering markets. Since the expected returns on the favorites are higher than those on the longshots, bettors could make profits by wagering on favorites rather than on longshots. Although recent papers³⁵ do not empirically support the favorite-longshot bias in the US and other countries, a great number of theories have been

another strategies based on their finding that late bets contain information about the horses' eventual ranking. The more informed bettors tend to make wagers at the end of the wagering period since an early bet would adversely affects the parimutuel odds. In recent years, the frequent use of those strategies has reduced the effectiveness significantly.

³³ Three empirical regularities are summarized in Ottaviani and Sorensen (2005c): the puzzle of early betting, late informed betting, and favorite-longshot bias.

³⁴ Hausch, Lo, and Ziemba (1994) summarized the empirical articles that provide evidence for favorite-longshot bias.

³⁵ Busche and Wall (2000), Busche and Hall (1988), Busche (1994), Gandar, Zuber, and Johnson (2001), Bjorkman and Bukszar (2003) are such examples.

formulated to explain this phenomenon³⁶. Basically, these theories can be categorized as two sets: risk-loving utility functions and misperceptions of probabilities. Based on the empirical tests of Snowberg and Wolfers (2007), the favorite-longshot bias is more likely driven by the misperceptions of probability, as suggested by the Prospect Theory, rather than by risk-loving utility.

2.3 Comparing Parimutuels and Traditional Insurance

Parimutuels aggregate purchases into a mutualized risk pool and reallocate risks among all participants; as indeed does insurance. In order to compare the cash flows of parimutuel participants and those of traditional insurance policyholders, we divide the event space into four disjoint outcomes:

- (1) No hurricane occurs
- (2) A hurricane hits areas outside the target area
- (3) A hurricane hits the target area but causes no damage to the policyholder's asset
- (4) A hurricane hits the target area and destroys the policyholder's asset

Cash flows of traditional policyholders and cash flows of parimutuel participants in these disjoint outcomes are depicted in Figure 2.1 and Figure 2.2, respectively. Premium

³⁶ Griffith (1949) ascribed to individuals' systematic undervaluation of favorites and overvaluation of longshots. Ali (1977) suggested that risk-seeking bettors are willing to accept lower returns for longshots. Hurley and McDonough (1995) proposed that a considerable track take reduces informed bettors' arbitrage opportunity, leading to few bets on favorites. Shin (1991;1992) argued that fixed-odds-betting bookmakers would prevent losses of the informed bettors. Ottaviani and Sorensen (2005c) noted that with a large number of bettors possessing some private information, posterior odds are more extreme than market odds, resulting in the bias. This paper also compared the merits and drawbacks among the theories described above.

denotes the premium of the traditional insurance; L is the potential loss; x^* is the parimutuel stake; I represents the indemnity. Thus, net indemnity is simply $(I - x^*)$.

For outcomes (1)-(4) of Figure 2.1, the net cash flows of traditional insurance are the same if $I=L$. The indemnity is received by policyholders only when the actual loss is incurred (i.e., outcome (4)), but the indemnity offset the actual loss³⁷. Thus, cash flows for policyholders of traditional insurance are negative (i.e., the insurance premiums) across four outcomes.

Cash flows of parimutuels are summarized as follows.

- In outcome (1), the cash flow is zero.

Stakes are returned to participants if no hurricane occurs.

- In outcome (2), the cash flow is a negative parimutuel stake.

Parimutuel stakes are paid out; no indemnity is obtained by the participants since the hurricane does not hit the target area.

- In outcome (3), the cash flow is the net indemnity, or the indemnity minus the parimutuel stake.

Parimutuel stakes are paid out; the indemnity is received since the hurricane hit the target area; no loss is incurred because the hurricane did not destroy the participant's asset.

- In scenario (4), the cash flow is net indemnity minus the loss.

³⁷ Based on Mossin's theorem, if proportional insurance is available at a fair price, without loading factor and default risk, full insurance is optimal for a risk-averse individual.

Parimutuel stakes are paid out; the indemnity is received since the hurricane hit the target area; the loss comes from the hurricane's destruction of the participant's asset.

A comparison of cash flows between parimutuel participants and insurance policyholders indicates that in outcomes (1) and (3), parimutuels are better than traditional insurances, but in outcomes (2) and (4), there is no monotone relation. For outcomes (1) and (3), this result is straightforward. The cash flows of parimutuel participants are zero in outcome (1) and positive in outcome (3)³⁸, respectively, whereas the cash flows of insurance policyholders are both negative. For outcome (2), the net cash flow depends on the relative size of the parimutuel stake and the insurance premium. If the insurance premium is greater than the parimutuel stake, parimutuels are better. For outcome (4), parimutuels are better whenever the insurance premium is greater than the loss minus the net parimutuel indemnity, i.e., $premium > L - (I - x^*)$. We use the following two examples to illustrate the comparison of cash flows in outcomes (2) and (4). In the first example, suppose that the probability of a hurricane's occurrence is very small and the insurance premium is less than the parimutuel stake³⁹. The parimutuel is worse than the traditional insurance under outcomes (2) and (4). In the second example, suppose the probability of a hurricane's occurrence is high enough so that the insurance premium is greater than the parimutuel stake⁴⁰; the parimutuel is better than the traditional insurance under outcome (2). Furthermore, if the probability of a hurricane's

³⁸ Net indemnity is positive; otherwise, nobody has the incentive to purchase insurance.

³⁹ As shown in Section 4, parimutuel stakes are not affected by the probability of a hurricane's occurrence, but the insurance premium decreases with the lower probability.

⁴⁰ Similar to footnote 14, parimutuel stakes are not affected by the probability of a hurricane's occurrence, but the insurance premium increases with the higher probability

occurrence is higher such that the insurance premium is greater than the loss minus net indemnity, i.e., $premium > L - (I - x^*)$, the parimutuel is better in all outcomes.

2.4 Optimal Parimutuel Stakes

Suppose a risk-averse individual participates in the parimutuel mechanism to hedge against his potential loss from a hurricane in area s without transaction fees and taxes. We will refer to such hedging as “parimutuel insurance”. W is the initial wealth of the individual, L is the loss that the individual will bear if a hurricane hits area s where his asset is located, p_h is the probability of the occurrence of a hurricane, p_s is the conditional probability of a hurricane hitting area s given that it actually occurs, p_i is the conditional probability the hurricane destroys the individual’s asset in area s given that it hits area s . Here, all probabilities and conditional probabilities are assumed to be objective and are known to the individual. “ x ” is the stake that the individual places to hedge against the potential damage of his asset in area s under the parimutuel mechanism. $m_s(x)$ is the total stakes collected from those who purchase parimutuel insurance on area s while $M(x)$ represents the total stakes of individuals across all areas.

Parimutuels define the payout over an exhaustive set of outcomes. To satisfy this property, the event space is defined to include a set of counties where the first landfall may occur, plus a null event where no hurricane occurs in any of the counties. If the

hurricane does not occur, the stakes gathered are distributed proportionally to each participant based on the initial stake⁴¹.

We further define the odds and their relation with the payoff to parimutuel participants if the hurricane hits area s . The final odds against area s and the total stakes on areas outside area s over the total stakes on area s are denoted as $O_s(x)$

$$O_s(x) = \frac{M(x) - m_s(x)}{m_s(x)} = \frac{M(x)}{m_s(x)} - 1$$

The payoff of the stake is as follows:

$$\text{payoff}_s(x) = \begin{cases} \frac{M(x)}{m_s(x)} & \text{if the hurricane hits area } s \\ 0 & \text{o.w.} \end{cases}$$

The ratio of the total stakes in all areas to the stakes in area s is simply the payoff if the hurricane hits area s . Thereby, the relation between the odds against area s and the payoff

when area s is hit can be exhibited as $O_s(x) = \frac{M(x)}{m_s(x)} - 1 = \frac{1}{P_s(x)} - 1$, where $P_s(x)$ is the

price of parimutuel insurance in area s , which is a function of x since the price alters as the individual places his stake. Furthermore, given the individual's stake on area s , x , the net indemnity of the parimutuel stake if the hurricane hits area s , is denoted as $NI_s(x)$. The relations among the net indemnity, the odds, the payoff, and the price of parimutuel claim

are thus shown as $NI_s(x) = O_s(x) \cdot x = \left(\frac{M(x)}{m_s(x)} - 1 \right) \cdot x = \left(\frac{1}{P_s(x)} - 1 \right) \cdot x$

⁴¹ This setting is different from that in the HuRLO market, where the exhaustive and disjoint outcomes include 78 regions with a potential hurricane threat and one “no landfall” outcome. Participants can also place stakes on the “no landfall” outcome. If the no hurricane makes landfall during a year, participants who place stakes on “no landfall” share all stakes based on their initial stakes.

Our goal is to obtain the optimal parimutuel stake that the individual chooses during the wagering period. In this section, when an individual places this stake, the stakes of other participants across all areas are assumed to have been revealed to the public so that the individual has perfect information on odds across all areas prior to his decision. This assumption is equivalent to a situation where the individual places his stake at the end of the wagering period after all other participants have placed their stakes. Although this assumption seems to be unrealistic, we start with a simple case. The functional forms of $m_s(x)$ and $M(x)$ are further specified as $m_s(x)=m_0+x$, $M(x)=M_0+m_0+x$, where m_0 denotes the total stakes on area s placed by other participants, and M_0 denotes the total stakes outside area s . In order to derive the optimal stake, the individual maximizes his own expected utility function:

$$\begin{aligned}
& \underset{x}{Max} \quad E[U(\tilde{W})] \\
& = (1 - p_h) \cdot U(W - x + \frac{M(x)}{M(x)} \cdot x) + p_h \cdot (1 - p_s) \cdot U(W - x) \\
& + p_h \cdot p_s \cdot (1 - p_i) \cdot U(W - x + \frac{M(x)}{m_s(x)} \cdot x) + p_h \cdot p_s \cdot p_i \cdot U(W - L - x + \frac{M(x)}{m_s(x)} \cdot x) \\
& = (1 - p_h) \cdot U(W) + p_h \cdot (1 - p_s) \cdot U(W - x) \\
& + p_h \cdot p_s \cdot (1 - p_i) \cdot U(W + O_s(x) \cdot x) + p_h \cdot p_s \cdot p_i \cdot U(W - L + O_s(x) \cdot x)
\end{aligned}$$

Yielding a F.O.C.

$$\frac{1 - p_s}{p_s} \cdot \frac{U'(W - x)}{(1 - p_i) \cdot U'(W + O_s(x) \cdot x) + p_i \cdot U'(W + O_s(x) \cdot x - L)} = \frac{\partial(NI_s(x))}{\partial x}$$

It is straightforward to confirm that the second order condition is negative⁴², and that x^* satisfying F.O.C. is a global maxima, and hence, the optimal parimutuel insurance stake

for the individual. The F.O.C. can also be presented as:
$$\frac{\partial x}{\partial NI_s(x)} = \frac{p_s}{1-p_s} \frac{E_i[U'(W_s)]}{U'(W_{ns})},$$

where W_{ns} is the individual's wealth when a hurricane hits an area outside area s , and W_s is the individual's wealth when a hurricane hits area s no matter whether it causes damage to the individual's asset in area s or not. If the individual's stake does not influence the odds against area s ⁴³, the left hand side of the F.O.C. represents the reciprocal of odds against area s (or the ratio of the conditional probability of a hurricane hitting area s to the conditional probability of a hurricane not hitting area s), i.e.

$$\frac{\partial x}{\partial NI_s(x)} = \frac{1}{O_s(x)} \text{ (or } \frac{p_s}{1-p_s} \text{) if } \frac{\partial O_s(x)}{\partial x} \approx 0.$$
 Replacing the left hand side of F.O.C. with

the relative conditional probabilities, the F.O.C. becomes: $E_i[U'(W_s)] = U'(W_{ns})$.

The result shows that the optimal stake will equate the marginal cost of a net payoff in

state "s", $\frac{\partial x}{\partial NI_s(x)}$, with the ratio of the expected marginal utilities in the "payoff" state,

"s" and the "no payoff" state, "ns". The net indemnity in state s depends on the odds against area s . If the impact of the individual's stake on area s is trivial, or equivalently, if the odds against area s are hardly influenced by the individual's stake, the relative marginal utility is simply the reciprocal of the odds against area s , or the ratio of the conditional probability of a hurricane hitting area s and the conditional probability of a hurricane not hitting area s . In this case, the optimal stake can be solved by equating the

⁴² The signs of the first and the second derivatives of the net indemnity if the hurricane hits area s with respect to the parimutuel stake are proven in section 2.10.

⁴³ Generally, this condition is true if the market is comparatively well developed or the mutualized risk pool is relative large.

expected marginal utility in the payoff state, “s”, with the marginal utility in the “no payoff” state, “ns”.

Basis risk is reflected in the expected marginal utility when a hurricane hits area s , i.e., $E_i[U'(W_s)]$, of the F.O.C. If the conditional probability of a hurricane destroys the individual's asset in area s , given that it hits area s , is further set to be one, or equivalently, when a hurricane hits the target area, all assets in the area will be destroyed, the optimal stake choice for the individual in the parimutuel is the same as the optimal amount of insurance in traditional insurance. Thus, parimutuel insurance can be differentiated from traditional insurance by the basis risk that the individual will encounter.

The optimal parimutuel stake derived by the F.O.C. for the individual is determined by six factors, i.e., $x^*(W, m_0, M_0, p_s, p_b, L)$:

- The initial wealth of the individual, the stakes on area s by other bettors
- The stakes outside area s
- The conditional probability of area s being hit by a hurricane if the hurricane should occur
- The conditional probability of the individual's asset being destroyed if a hurricane hits area s
- The potential loss

It is of interest that the probability of a hurricane's occurrence, p_h , does not have any impact on the optimal parimutuel stake. The rationale is intuitive: the redistribution of parimutuel stakes among participants occurs only when a hurricane hits one of the S

areas. If no hurricane occurs, these stakes are given back to the participants without any transaction fee or tax. Moreover, the final odds against area s play a key role in determining the optimal stake. In parimutuels, odds, which are crucial to determine the payoff, fluctuate through time until the close of the wagering period. As an additional stake is placed on one area, the odds against all areas change accordingly. The individual thus has an incentive to postpone his stakes until more information about the odds are revealed to decide his optimal stake⁴⁴.

The above optimal decision is treated as a static problem. However, the parimutuel market evolves over the year with the odds changing. Thus, this should be a dynamic hedging problem. The static optimal hedge will be achieved when trading closes based on available information. Furthermore, how to use our results to implement dynamic hedging in the HuRLOs market is shown as follows. If the impact of an individual stake on odds is negligible, the optimal stake can be determined by the following equation:

$$\frac{1 - p_s}{p_s} \cdot \frac{U'(W - x^*)}{(1 - p_i) \cdot U'(W + O_s(x) \cdot x^*) + p_i \cdot U'(W + O_s(x) \cdot x^* - L)} = O_s(x), \text{ which is}$$

modified from the F.O.C. As can be seen, the optimal parimutuel stake is a function of several parameters, including the odds and the conditional probability of area s being hit.

In trading the HuRLOs, the investors know the odds, $O_s(x)$ ⁴⁵ (from market-based

⁴⁴ The timing of bettors' wagers, as indicated in Ottaviani and Sorensen (2005c), is determined by two opposing forces: the bettor's market power and the bettor's concern about information revelation. In a simultaneous-move game, bettors holding sizeable wagers and common information would bet early to prevent from adverse impact on odds while many small bettors with private information would place a late bet to conceal their private information and to grasp more information on the odds from other bets. Under the parimutuel mechanism, informed bettors prefer to postpone their wager until the last minutes because all bets are executed at the same final price.

⁴⁵ $O_s(x) = \frac{1}{\tilde{p}_s} - 1$, where $\tilde{p}_s = \frac{\hat{p}_s}{1 - \hat{p}_{nh}}$, \hat{p}_s is the market-based probability of the first hurricane making landfall in region s , \hat{p}_{nh} is the market-based probability of no hurricane making landfall in any of

probabilities) and the conditional probabilities, p_s ⁴⁶ (from forecast-based probabilities) at any point in time. Both market-based probabilities and forecast-based probabilities in all covered regions are public information for investors and are available on the WRS website. Thus, given the optimal stake rule, hedgers can actually determine their hedging position with available information.

2.5 Equilibrium of Parimutuel Stakes

In this section, we analyze a risk-averse representative agent's optimal stake decision under the parimutuel mechanism. In this special case, S areas are at risk to be hit by a hurricane. Suppose all residents participate in the parimutuel insurance and own assets with the same market value in one of the S areas. There are n residents in each area. The hurricane hits each area with an equal probability. An individual in area s is taken to be a representative agent who has W dollars for his initial wealth, places the stake worthy of x in parimutuels, and losses L dollars if a hurricane hits his asset in area s . Since p_s are the same across all areas, $p_s = 1/S$. When a hurricane hits area s , the payout to the representative agent is the ratio of the total stakes to the number of residents in the area, i.e., $(S \cdot n \cdot x)/n = S \cdot x$. If no hurricane occurs, the total stakes are returned proportionally to the initial stake.

the covered regions. The market-based probabilities for all covered regions are available on the WRS trading website.

⁴⁶ $p_s = \frac{\bar{p}_s}{1 - \bar{p}_{nh}}$, where \bar{p}_s is the forecast-based probability of the first hurricane making landfall in region s , \bar{p}_{nh} is the forecast-based probability of no hurricane making landfall in any of the covered regions. The forecast-based probabilities for all covered regions are available on the WRS trading website.

The objective is to maximize the expected utility of the representative agent:

$$\begin{aligned} & \underset{x}{Max} \ E[U(\tilde{W})] \\ & = (1 - p_h)U(W) + p_h \cdot (1 - p_s)U(W - x) + p_h \cdot p_s \cdot (1 - p_i) \cdot U(W + (S - 1) \cdot x) \\ & + p_h \cdot p_s \cdot p_i \cdot U(W + (S - 1) \cdot x - L) \end{aligned}$$

Differentiating the objective function with respect to x yields the F.O.C.:

$$\begin{aligned} & \frac{\partial E(\tilde{W})}{\partial x} \\ & = -p_h \cdot \frac{S-1}{S} \cdot U'(W - x) + p_h \cdot \frac{S-1}{S} \cdot (1 - p_i) \cdot U'(W + (S - 1) \cdot x) \\ & + p_h \cdot \frac{S-1}{S} \cdot p_i \cdot U'(W - L + (S - 1) \cdot x) = 0 \\ & \Rightarrow U'(W - x^*) = (1 - p_i) \cdot U'(W - x^* + \underbrace{S \cdot x^*}_{\geq 0}) + p_i \cdot U'(W - x^* + S \cdot x^* - L)^{47} \end{aligned}$$

For the F.O.C. to hold with $S \cdot x^* \geq 0$ & $U'' < 0$, then $S \cdot x^* - L \leq 0$, (or equivalently, $x^* \leq L/S$), must be satisfied. Thus a parimutuel will, in general, provide a payout to the representative agent that is less than the potential loss. This result rests on the presence of basis risk. The parimutuel pays on the occurrence of loss in area s . However, conditional on area s being hit, the probability that the stakeholder suffers a loss, p_i , is less than one. Thus, the parimutuel provides a windfall gain to an individual even if he/she suffers no loss. In effect, the parimutuel introduces background risk. We know, in general, that the presence of background risk will upset the normal optimality condition for full (actuarially fair) insurance. Only in the special case where $p_i=1$; (equivalently $n=1$ or only one stakeholder in area s), will the F.O.C. show that a full

⁴⁷ The negative sign of the second derivative is easily verified.

indemnity, $x^* = L/S$ is optimal, consistent with Mossin's theorem⁴⁸. Thus, parimutuel insurance intrinsically leads to underinsurance even in the case of no transaction fee or tax.

However, for the general case where $p_i < 1$, the parimutuel offers a payout on the hurricane hitting the target area s whether or not the individual actually has a loss. In contrast, the mutual insurance rules out the possibility of the windfall gain, and hence the policyholder's decision only depends on balancing marginal utilities in two states: when the hurricane hits area s and when the hurricane hits other areas, i.e.,

$U'(W - x^*) = U'(W - L + (S - 1) \cdot x^*)$. A risk-averse mutual insurance policyholder will choose full insurance, i.e., $x^* = L/S$, since the insurance premium genuinely reflects the expected insured losses.

We further explore the comparative statics on the equilibrium of the parimutuel stake regarding the underlying four parameters; $x^*(W, p_s, p_i, L)$ ⁴⁹. The summary of our analysis is as follows. The sensitivities of the equilibrium of the parimutuel stake with respect to the loss caused by a hurricane, with respect to the conditional probability of a hurricane hitting the target area given that it occurs, and with respect to the conditional probability of an individual's asset being destroyed given that the hurricane hits the target area, are all positive, i.e., $\partial x^* / \partial L > 0$, $\partial x^* / \partial p_s > 0$, and $\partial x^* / \partial p_i > 0$.

⁴⁸ Mossin's Theorem is often considered the cornerstone result of insurance economics. It is attributed to Mossin (1968).

⁴⁹ For the same reason as was stated in section 2.4, the likelihood of the occurrence of a hurricane does not affect the equilibrium of stake. The redistribution of overall stakes is only triggered by a hurricane's hitting one of the S areas.

The relation between the parimutuel stake and the conditional probability of a hurricane hitting area s suggests that the sensitivity of the equilibrium of the parimutuel stake with respect to the number of hurricane-prone areas is negative, i.e., $\partial x^* / \partial S < 0$. The basic logic is simply balancing between the expected marginal utilities for two states: when a hurricane hits the target area and when a hurricane hits areas outside the target area. First, *ceteris paribus*, an increase in the number of hurricane-prone areas decreases the expected marginal utility when a hurricane hits the target area by increasing the payoff to the target area. The equilibrium parimutuel stake is then reduced, not only to decrease the payout when a hurricane hits areas outside the target area, but also to reduce the net indemnity when a hurricane hits the target area. Through this adjustment, the expected marginal utilities in these two states are equalized.

Similar to the previous analysis but with a reverse direction, an increase in the potential loss (or the conditional probability of the asset being destroyed by a hurricane given that it hits the insured area), *ceteris paribus*, raises the expected marginal utility when a hurricane hits the target area. In order to balance off the relative expected marginal utilities, the equilibrium of the parimutuel stake moves upward both to enhance the net indemnity when a hurricane hits the target area and to reduce the stake payout when a hurricane hits the areas outside the target area.

Nevertheless, the sensitivity of the equilibrium parimutuel stake with respect to the initial wealth is ambiguous. We derive the condition under which the sign of the sensitivity is positive:

$$\frac{\partial x^*}{\partial W} > 0 \Leftrightarrow U''(W - x^*) < (1 - p_i) \cdot U''(W + (S - 1) \cdot x^*) + p_i \cdot U''(W + (S - 1) \cdot x^* - L).$$

The rationale for this result is also similar. As the initial wealth increases, expected marginal utilities in both states decline accordingly. If the above condition holds, the marginal utility when a hurricane hits areas outside the target area is reduced for a greater amount than the expected marginal utility when a hurricane hits the target area. To balance between expected marginal utilities in these two states, the equilibrium of the parimutuel stake has to adjust upward. Section 2.11 provides the detailed derivations of the comparative statics and the determination of their signs for reference.

2.6 Pros and Cons of Parimutuel Insurance

Parimutuels have several merits that can potentially ameliorate or even overcome the recent obstacles that are encountered by insurers. Here is our summary:

- The insurer takes no risk.

This is the most critical advantage of parimutuels. The insurer acts as a bookmaker and redistributes money among policyholders. Parimutuels add no financial capacity constraints to insurers.

- Liquidity is enhanced.

In trading catastrophe-linked derivatives⁵⁰, traders have to find a counterparty who is willing to take the underlying risk on the other side. If trading parties do not match in a market, high bid-ask spreads or thin trading volume will appear. Catastrophe

⁵⁰ CAT-linked securities, such as Industry Loss Warranty (ILW), CAT options, and CAT bonds first appeared in the capital markets in the 1980s and 1990s. In addition to traditional reinsurance contracts, these securities together with Sidecars⁵⁰, provide risk-transferring alternatives in ameliorating CAT shocks within the economy.

futures⁵¹ are such an example. However, in parimutuels, policyholders can bet on any adverse outcome in the risk pool without insurers' matching offers.

- There is less default risk.

In parimutuels, all indemnity payouts come from the stakes of the participants. Since all stakes are collected before adverse events and insurers simply redistribute the stakes, policyholders and insurers have less incentive to default on the transaction.

- Windfall gain is possible in parimutuels.

Parimutuels pay out when a hurricane hits the target area whether or not the policyholder actually has a loss. When the hurricane hits the target area without destroying the participant's asset, the participant shares the total stakes with other target bettors, even if he does not bear any loss.

- Costs are reduced for insurers

Traditional insurers must have expertise to estimate the probability of a hurricane's occurrence, the probability of a hurricane hitting each area, and the potential losses for policyholders. Large insurers usually hire hurricane modeling firms⁵² or build a team of experts from different fields⁵³ to approximate and hedge the risk they take.

However, parimutuel insurers bear no risk, thus they are not required to manage the catastrophic risk.

⁵¹ Catastrophe Futures were introduced by the Chicago Board of Trade (CBOT) in 1992 after Hurricane Andrew.

⁵² The scientific risk analysis and quantitative risk estimates of the catastrophe damage is developed by catastrophe-modeling firms. Those firms build catastrophe models, which use meteorology, engineering, and insurance underwriting data to estimate damage in different areas. Input information is based on historical tropical storms, building construction, and the impact on various structures under different wind speeds. Today, the three leading proprietary catastrophe modeling firms are Risk Management Solutions (RMS), AIR Worldwide, and EQECAT.

⁵³ Include meteorology, engineering, and insurance underwriting.

- The contracts are more flexible.

Traditional insurance generally specifies a fixed period, like annual or semiannual, as the insured period so that the premiums collected are enough to cover the potential indemnity. Parimutuels have more flexibility in setting insured periods. For instance, we can specify the insured period to be hurricane-prone seasons based on historical data. Policyholders do not have to pay premiums for the whole year.

- Parimutuels ameliorate information asymmetry.

In order to deal with adverse selection, insurers have to design mechanisms⁵⁴ to distinguish clients with different risk types. In parimutuels, however, payouts depend only on the area hit by the event not upon participant characteristics. Similarly, payouts are independent of actions of participants (other than choice of stake). Thus moral hazard is avoided.

On the other hand, parimutuel insurance has several drawbacks.

- Parimutuel participants are underinsured.

In response to basis risk, parimutuel participants would naturally choose the optimal parimutuel stake that provides less than full coverage should the participant suffer a loss.⁵⁵ In contrast, policyholders of traditional insurance would choose full insurance.

- Participants do not know the payoffs (or prices) when placing stakes.

⁵⁴ The self-selection mechanism, in which insurers offer a menu of insurance contracts with various prices and quantities so that insurers can distinguish the risk types of policyholders by observing their choices. Related literature include Rothschild and Stiglitz (1976), Spence (1978), Dionne (1983), Kunreuther and Pauly (1985), Dionne and Doherty (1994), etc.

⁵⁵ The indemnity is insufficient to pay for the actual loss.

The potential payoffs are determined by the final distribution of the total stakes, which will not be revealed until the end of the wagering period. This feature can be abated by postponing the wager as late as possible before the wagering period closes, but can not be eliminated.

- Late Information

Transactions are not allowed during the period between the end of the wagering period and the eventual outcome is realized. When adverse events against an individual's bets occur after the wagering period, it is impossible to withdraw his stakes. If the underlying risk is highly sensitive to updated information over time, participants could not react to incoming information immediately, thus having a lock-in risk. However, it is unlikely that insurer would underwrite at this time. Participants also have incentives to postpone the timing of purchasing parimutuel insurances in order to wait for the most updated information to be revealed⁵⁶. There is an offsetting effect, if there is secondary trade in parimutuel stakes after the wagering period closes. Since secondary trades will reflect new risk, then hedging strategies are possible at this late time.

- Basis Risk

When the probability of hurricane occurrence is low, the parimutuel stake could be higher than the traditional insurance premium. In addition, cash flows of parimutuels are less than traditional insurance when the hurricane hits areas other than the target area and when the hurricane destroys the assets of the participants.

⁵⁶ The late informed betting phenomenon, as described in Ottaviani and Sorensen (2005c): late bets contain a great deal of information about the horses' eventual ranking in the parimutuel game. Also documented in Asch, Malkiel, and Quandt (1982), Gandar, Zuber, and Johnson (2001), and Ottaviani and Sorensen (2005a).

- Parimutuels can only be applied to mutually exclusive events.

Parimutuel pricing principles restrict the outcomes to be collectively exhaustive and mutually exclusive. Collectively exhaustive property can be relaxed by including a no hurricane state, but a mutually exclusive condition can not be violated. In applying parimutuels, it is important to define the insured events to be mutually exclusive. In a HuRLOs scheme, although a hurricane could hit more than one of the covered areas, the trigger event is defined as the covered area first hit by a hurricane during a year.

2.7 The Role of Speculators

A useful of parimutuels is that one purchase on one outcome will improve liquidity for all other outcomes. Thus, adding speculators to our model may enhance liquidity.

Speculation could play a somewhat neutral role in our analysis if the speculators placed their bets across counties in direct proportion to the stakes placed by hedgers. But it is difficult to see what advantage speculators would derive from such a passive strategy. More likely, they would take advantage of perceived mispricing by placing bets on outcomes where the objective probabilities differ from market-based probabilities. This would, reduce (increase) the payoffs to hedgers in counties where speculators take net long (short) positions.

In our previous analysis, we assume that hedgers use the objective probability to estimate the likelihood that a hurricane makes landfall in a county. Laypersons have difficulty in predicting the probability that a hurricane makes landfall and have to resort to the objective probability that provided by meteorologists. The aggregate subjective

probability of the first landfall should thus approach the corresponding objective probability, supporting our usage of the objective probability instead of subjective probability in optimizing the utility. Moreover, the market-based probability simply reflects the relative demand in that county. Relative higher demand in the target county compared to the objective probability will raise the market-based probability of the target relative to other counties. In this case, the market-based probability of the target is higher than the objective probability of the target, whereas the market-based probability of other counties is lower than the objective probability of other counties. Speculators would thus arbitrage on the discrepancy between the objective probability and the market-based probability by placing more stakes in other counties except for the target. This behavior, through parimutuels, will decrease the market-based probability of the target and increase the market-based probability of other areas until reaching the equilibrium, where the market-based probability equals the objective probability for all counties. Thus, in the presence of speculators, our results in section 2.4 will be approaching the equilibrium.

If the market is in equilibrium, an additional stake on a specific outcome placed by a speculator will drive down the payoff of that outcome while push up the payoff of all other outcomes. Nevertheless, the expected payoff on each outcome is still kept at the same level as prior to the additional stake thanks to the parimutuel mechanism. Speculators can not make any profit by placing any bet that deviates from the equilibrium. Since speculators would not participate in the equilibrium, the assumption on the absence of speculators in our models is then justified.

2.8 Equivalent Transaction Costs of Traditional Insurance Relative to HuRLOs

If there were no transaction costs, the optimal hedge with a HuRLO would not provide full insurance because of the inherent basis risk. In contrast, with traditional (indemnity) insurance with no transaction cost, it is optimal to fully insure. However, with transaction costs, the choice between hedging with a HuRLO or with insurance, essentially boils down to a trade off between transaction costs and basis risk. Although we do not have estimates of the transaction costs of HuRLOs, we believe these to be very low relative to traditional insurance, and probably in the order of a 1-2%. In this section we would like to provide rough estimates of the tradeoff between transaction costs and basis risk. We will do so by estimating the optimal HuRLO stake for an assumed level of transaction cost. We will then estimate the transaction cost for a traditional insurance contract that equates the expected utility with optimal insurance to the expected utility yielded by the optimal HuRLO stake. If, this actual transaction costs for traditional insurance are higher (lower) than the utility equivalent level, it follows that the individual will be better (worse) off with the HuRLO than with traditional insurance.

Suppose that an individual possesses total assets with \$1,000,000 in value, including a house that is worth \$500,000. If hurricane strikes the area where the house located and destroys the house, he will suffer a loss of \$500,000. The utility of the individual can be represented by a simple power utility function with a constant relative risk aversion coefficient, a , i.e., $U(W) = \frac{1}{1-a} W^{1-a}$, where a is assumed to be 0.71. This risk aversion estimate should be reasonable based on the reviews of empirical findings in Chetty

(2006) and Cardenas and Carpenter (2007). The former summarizes 33 sets of estimates of labor supply elasticities and calculates the implied coefficients of relative risk aversion ranging from 0.15 to 1.78 with mean 0.71 while the latter review coefficients from risk experiments in various countries and finds estimates between 0.05 and 2.57. Palm Beach and Monroe, both located in Florida, are the two target areas that we will focus on to illustrate the equivalent transaction cost relative to HuRLOs. For each area, there exist two sets of probability: one is forecast-based probability and the other is market-based probability. From actual trading data on a specific date in 2009, the two sets of probabilities are ($p_h=0.0395$, $p_s=0.0228$, $p_i=0.1$) and ($p_h^m=0.2454$, $p_s^m=0.0595$, $p_i=0.1$) in Palm Beach and ($p_h=0.0395$, $p_s=0.0734$, $p_i=0.1$) and ($p_h^m=0.2454$, $p_s^m=0.0717$, $p_i=0.1$) in Monroe. If a hurricane hits an area, we suppose that there is a one-in-ten chance that houses within the area will be destroyed ($p_i=0.1$). Thus, the optimal stake and the maximal expected utility in the HuRLOs market can be obtained by first collecting the probability of hurricane occurring (p_h) and the conditional probability of hurricane striking a specific area (p_s) from the HuRLOs market, inputting them into the expected utility of the individual, and then exercising optimization. The expected utility is similar to that is shown in section 2.4 with the odds derived by the conditional probability of hurricane striking the area, $O_s=(1/p_s)-1$.

The maximal expected utilities empirically obtained from HuRLOs are used to derive the equivalent transaction cost of the traditional insurance contract by assuming the individual is fully-insured and partially-insured, respectively. If the individual is fully-insured, the equivalent transaction cost simply reflects the basis risk of the specific HuRLO; however, if the individual is allowed to be partially-insured, the equivalent

transaction cost balances the synthetic effects of the basis risk of the HuRLO with the individual's incentive to buy less than full insurance when insurance is costly.

Figure 2.3 shows the comparisons of the expected utilities for an individual between HuRLOs and traditional insurance in four cases with two dimensions, the area (Palm Beach/Monroe) and the type of probability (forecast-based/market-based), assuming no transaction cost in HuRLOs. For the same case, the left hand side exhibits the HuRLOs and the right hand side demonstrates the traditional insurance. Take the HuRLO in Palm Beach using forecast-based probabilities as an example. The optimal stake is \$1,785.1 and the maximal expected utility is 189.4939. In order to achieve this expected utility in traditional insurance contracts given the constraint that the individual is fully-insured, the equivalent transaction cost is 20.91%. Thus if the actual transaction cost for insurance is greater than 20.91%, the individual will be better off accepting the basis risk of the HuRLO than full indemnity insurance. But, of course, full insurance is sub optimal with transactions costs. When being partial-insured is allowed (the optimal amount of insurance and the equivalent transaction cost are determined simultaneously), the optimal amount of insurance is 33.55% and the equivalent transaction cost is 33.2%. This equivalent transaction cost is higher than that of the full-insured case because the individual would choose to purchase less insurance in the presence of transaction costs.

According to the statistics of premiums and losses of 2009 top 25 companies in property and casualty insurance industry, collected by NAIC (National Association of Insurance Commissioners), the ratio of losses to premiums is approximately 60% (See Table 1, this ratio is between 59.27% to 60.94% by different definitions of losses). Empirical transaction costs of traditional homeowners multiple peril insurance are

approximately 67% ($(1/60\%)-1$) of the expected loss, which include administrative, marketing and claims processing costs, taxes, and cost of capital. The equivalent transaction costs relative to HuRLOs in the four cases range from 33.2% to 43.2%, depending on transaction costs in HuRLOs (0%-2%). Thus, the actual transaction costs for traditional insurance (67%) are higher than the utility equivalent level (33.2%-43.2%). This result suggests that HuRLOs, in spite of the inherent basis risk, provide a more effective hedging mechanism than traditional insurance for homeowners because the transaction cost of HuRLOs is very low (less than 2%) relative to traditional insurance.

2.9 Conclusions

This paper compares parimutuels to insurance contracts in a market where risk-averse people seek to hedge. We construct two models where an individual places a parimutuel stake to hedge against potential catastrophes. In the first model, an optimal stake choice is obtained when the total stakes on the target area placed by other participants, and the total stakes outside the target area, are exogenous. The optimal stake can be obtained by equating the marginal cost of a net payoff with the ratio of the expected marginal utilities in the payoff state and the no payoff state. When the odds and the conditional probability of a hurricane hitting the target areas are available, we derive the dynamic optimal hedge rule by the first order condition. In the next model, the equilibrium of the parimutuel stake is derived based on a representative agent's optimal choice. If there is no transaction fee and tax, parimutuel insurance intrinsically leads to underinsurance due to the basis risk, a result that is inconsistent with traditional insurance where the decision

maker has background risk. Furthermore, the sensitivities of the equilibrium of the parimutuel stake with respect to the potential loss, the conditional probability of a hurricane hitting the target area if it occurs, and the conditional probability of an individual's asset being destroyed if a hurricane hits the target area, are all positive; however, the sensitivity of the equilibrium with respect to the initial wealth is ambiguous.

WRS invented an option, called HuRLOs, which embodies the concept of parimutuel insurance. Issuers of HuRLOs act as a bookmaker and do not bear the underlying risk, but participants who intend to hedge will have inbuilt basis risk compared to traditional insurance with no transaction cost. However, in the presence of transaction costs, the choice between hedging with a HuRLO or with insurance, essentially boils down to a trade off between transaction costs and basis risk. We derive the equivalent transaction costs such that the expected utility with optimal insurance equates the expected utility yielded by the optimal HuRLO stake. The actual transaction cost for traditional homeowners multiple peril insurance is approximately 67%, higher than the utility equivalent level (33.2%-43.2%) implied by the HuRLOs with minor transaction cost. This empirical result suggests that homeowners who would like to hedge against hurricane risk would be better off with HuRLOs than with traditional insurance because in practice, the transaction cost of HuRLOs is very low relative to traditional insurance.

In this paper, the parimutuel market is analyzed focusing on hedgers. Speculators simply arbitrage on the discrepancy between the forecast-based probability and the market-based probability, and thus have no role to play in equilibrium. However, speculators could deviate from this simple arbitrage, because of irrational behaviors affected by psychological factors, or because they have different belief about the

objective probability. Empirical anomalies, such as the favorite-longshot bias, may result from these behaviors of speculators. Taking the actions of speculators into consideration may change the optimal stake choice for hedgers in parimutuels.

Appendix

2.10 The Derivatives of the Net Indemnity with Respect to the Parimutuel Stake

$$\begin{aligned}
& \frac{\partial E[U(\tilde{W})]}{\partial x} \\
&= \frac{1 - p_s}{p_s} \cdot \frac{U'(W - x)}{(1 - p_i) \cdot U'(W + O_s(x) \cdot x) + p_i \cdot U'(W + O_s(x) \cdot x - L)} \\
&= \frac{\partial(O_s(x) \cdot x)}{\partial x} \\
&= \frac{\partial(M_s(x))}{\partial x} \\
&= \frac{\partial(O_s(x))}{\partial x} \cdot x + O_s(x) \\
&= \frac{M_0}{m_0 + x} \cdot \frac{m_0}{m_0 + x} \\
& \frac{\partial[O_s(x) \cdot x]}{\partial x} = \frac{m(x) - M(x)}{[m(x)]^2} \cdot x + \frac{M(x)}{m(x)} - 1 = \frac{M(x) - m(x)}{m(x)} \cdot \frac{m(x) - x}{m(x)} \\
&= \frac{M_0 \cdot m_0}{(m_0 + x)^2} > 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 [O_s(x) \cdot x]}{\partial x^2} &= \frac{-2 \cdot [m(x) - M(x)]}{[m(x)]^3} \cdot x + 2 \cdot \frac{m(x) - M(x)}{[m(x)]^2} \\
&= -2 \cdot \frac{M(x) - m(x)}{[m(x)]^2} \cdot \frac{m(x) - x}{m(x)} = -2 \cdot \frac{M(x) - m(x)}{[m(x)]^3} \cdot (m(x) - x) \\
&= \frac{-2 \cdot M_0 \cdot m_0}{(m_0 + x)^3} < 0
\end{aligned}$$

2.11 The Comparative Statics of the Equilibrium Parimutuel Stake

Rearranging from the F.O.C., we have the following formula:

$$\begin{aligned}
&f(W, x^*, p_i, S, L) \\
&= U'(W - x^*) - (1 - p_i) \cdot U'(W + (S - 1) \cdot x^*) - p_i \cdot U'(W - L + (S - 1) \cdot x^*) = 0
\end{aligned}$$

The comparative static of the equilibrium of pari-mutuel stake with respect to four parameters in our model can be derived by Implicit Function Theorem:

$$\frac{\partial x^*}{\partial S} = -\frac{\frac{\partial f}{\partial S}}{\frac{\partial f}{\partial x^*}}, \quad \frac{\partial x^*}{\partial p_i} = -\frac{\frac{\partial f}{\partial p_i}}{\frac{\partial f}{\partial x^*}}, \quad \frac{\partial x^*}{\partial L} = -\frac{\frac{\partial f}{\partial L}}{\frac{\partial f}{\partial x^*}}, \quad \frac{\partial x^*}{\partial W} = -\frac{\frac{\partial f}{\partial W}}{\frac{\partial f}{\partial x^*}}$$

Implicitly differentiating f with respect to the x^* , p_i , S , L , and W yields the following equations with their signs:

$$\begin{aligned}
\frac{\partial f}{\partial x^*} &= -U''(W - x^*) - (1 - p_i) \cdot (S - 1) \cdot U''(W + (S - 1) \cdot x^*) \\
&- p_i \cdot (S - 1) \cdot U''(W - L + (S - 1) \cdot x^*) > 0
\end{aligned}$$

$$\frac{\partial f}{\partial p_i} = U'(W + (S-1) \cdot x^*) - U'(W - L + (S-1) \cdot x^*) < 0$$

$$\Rightarrow \frac{\partial x^*}{\partial p_i} = - \frac{\frac{\partial f}{\partial p_i}}{\frac{\partial f}{\partial x^*}} > 0 \Leftrightarrow \frac{\partial x^*}{\partial n} < 0 \left(\because p_i = \frac{1}{n} \right)$$

$$\begin{aligned} \frac{\partial f}{\partial S} &= -(1-p_i) \cdot x^* \cdot U''(W + (S-1) \cdot x^*) - p_i \cdot x^* \cdot U''(W - L + (S-1) \cdot x^*) \\ &= -x^* \cdot U''(W + (S-1) \cdot x^*) + p_i \cdot x^* \cdot \underbrace{\left[U''(W + (S-1) \cdot x^*) - U''(W - L + (S-1) \cdot x^*) \right]}_{>0} > 0 \end{aligned}$$

$$\Rightarrow \frac{\partial x^*}{\partial S} = - \frac{\frac{\partial f}{\partial S}}{\frac{\partial f}{\partial x^*}} < 0 \Leftrightarrow \frac{\partial x^*}{\partial p_s} > 0 \left(\because p_s = \frac{1}{S} \right)$$

$$\frac{\partial f}{\partial L} = p_i \cdot U''(W - L + (S-1) \cdot x^*) < 0$$

$$\Rightarrow \frac{\partial x^*}{\partial L} = - \frac{\frac{\partial f}{\partial L}}{\frac{\partial f}{\partial x^*}} > 0$$

$$\begin{aligned} \frac{\partial f}{\partial W} &= U''(W - x^*) - (1-p_i) \cdot U''(W + (S-1) \cdot x^*) - p_i \cdot U''(W - L + (S-1) \cdot x^*) \\ &= \underbrace{\left[U''(W - x^*) - U''(W + (S-1) \cdot x^*) \right]}_{<0} \\ &\quad + p_i \cdot \underbrace{\left[U''(W + (S-1) \cdot x^*) - U''(W - L + (S-1) \cdot x^*) \right]}_{>0} \end{aligned}$$

$$\Rightarrow \frac{\partial x^*}{\partial W} = - \frac{\frac{\partial f}{\partial W}}{\frac{\partial f}{\partial x^*}} > 0 \Leftrightarrow \frac{\partial f}{\partial W} < 0$$

Since the sign of $\frac{\partial x^*}{\partial W}$ can not be determined, we further analyze under the following

condition the sign is positive.

$$\frac{\partial x^*}{\partial W} > 0 \Leftrightarrow$$

$$U''(W - x^*) - (1 - p_i) \cdot U''(W + (S - 1) \cdot x^*) - p_i \cdot U''(W - L + (S - 1) \cdot x^*) < 0$$

$$\therefore \frac{\partial x^*}{\partial W} > 0 \Leftrightarrow U''(W - x^*) < (1 - p_i) \cdot U''(W + (S - 1) \cdot x^*) + p_i \cdot U''(W - L + (S - 1) \cdot x^*)$$

Chapter 3

Mitigating Losses from Climate Change through Insurance

3.1 Introduction

Natural disasters have caused more severe insured losses to property in recent years than in the past. The losses caused by great natural disasters have increased dramatically in recent years, especially after 1990. Based on the recent book⁵⁷, catastrophes also have a more devastating impact on insurers over the past 15 years than in the entire history. Before 1988, the worldwide insured losses from natural disasters are rarely greater than \$10 billion dollars. Nonetheless, after 1990, there is a radical increase in insured losses. The representative catastrophe events lead to those losses include Hurricane Ike in 2008, which lead to insured losses of \$16 billion, Hurricane Andrew in 1992, which cost insurers \$23.2 billion, and Hurricane Katrina in 2005, which caused up to \$46.3 billion in damage. Higher population density along the coast and increasing development in hazard-prone areas further exacerbate the situation⁵⁸. These facts manifest a radial change in the scale and rhythm of catastrophes, which in turn forecast that the future catastrophic losses could rise dramatically.

⁵⁷ For more detailed figures, refer to Kunreuther and Michel-Kerjan (2009)

⁵⁸ Based on Changnon (2003), Muir-Wood et al. (2006), Miller et al. (2008), and Crompton and McAneney (2008), the major source of rising damages caused by natural disasters comes from the concentration of human settlements and wealth in hazard-prone areas.

Climate change also potentially amplifies catastrophe risks and challenge current risk management strategies by raising the magnitude and frequency of certain extreme events. This phenomenon is pronounced in the highly exposed areas, such as US Gulf Coast and the Caribbean, rising hazards are likely to reduce the effectiveness of current risk mitigation and threaten insurability. Based on the conclusion of Intergovernmental Panel on Climate Change (IPCC (2007)), increasing concentrations of greenhouse gases, primarily human-induced, are the major source to warm the atmosphere and oceans⁵⁹. The elevated temperatures raise sea level by expanding ocean water, increasing the rate at which glaciers and ice sheets melt ice into the oceans⁶⁰. The number, track, rainfall quantity, and intensity of tropical cyclones might also change with global warming, driving more intense and frequent natural disasters. Sea level rise and potential stronger storms pose a more intensive threat to the economy, particularly in the coastal areas. The recent report⁶¹ on coastal flooding management suggests that over the next 100 years, higher sea level provides an elevated base for storm surges to build upon and diminishes the rate at which low-lying areas drain, thereby extending coastal inundation from rainstorms. Greater flood damages are also driven by increases in shore erosion,

⁵⁹ Human behaviors, including the burning of fossil fuels, deforestation, and other land use changes, contribute to the emission of carbon dioxide and other greenhouse gases, such as methane, which have accumulated in the atmosphere since late 19th century. Greenhouse gases trap heat more easily, resulting in higher surface air temperature. IPCC predicts that global average surface temperatures will increase 1.1°C~2.9°C under a low emission scenario and 2.4°C ~6.4°C under a high emission scenario. Stern (2007) also suggests that positive feedback mechanisms of climate change, such as releases of methane resulting from melting of permafrost and a reduced uptake of carbon that caused by shrinking Amazon forest, may amplify greenhouse gas concentrations and lead to global warming that is more severe than anticipated by climate models.

⁶⁰ IPCC (2007) predicts that sea level may rise 0.2~0.6 meter by 2100. Although the procedures may take several centuries, an irreversible melting of Greenland ice or collapse of the West-Antarctic Ice Sheet gives rise to a substantial increase in global sea level rise, about 5 to 12 meters, as indicated in Rapley (2006) and Wood et al. (2006).

⁶¹ U.S. Climate Change Science Program “Coastal Sensitivity to Sea-Level Rise: A Focus on the Mid-Atlantic Region”

removing protective dunes, beaches, and wetlands and thus leaving previous protected properties closer to the water's edge.

Climate change alters the catastrophic risks by increasing existing risk over time. A crucial feature of climate change lies in its irreversibility⁶²: once the climate regime in an area has change, it is unlikely to restore to the original state. Failure to take the potential of climate change into account would lead insurers to underestimate insurance premiums and reserves for catastrophic risk exposures, causing higher possibility of bankruptcy or financial distress, which in turn affect insurability. The best way to handling effects of climate change for insurers is incorporating possible changes in weather extremes to assess and manage future catastrophic risks. Even through the nature of low-probability and uncertainties of theses events leads to difficulties in estimating the impact of climate change on natural disasters and their resulting damage⁶³, it is critical to establish a reasonable catastrophic risk model considering potential climate change and the associated uncertainties in order to help insurers enhance the accuracy of risk assessment, improve risk management, and further strengthen long-term sustainability against the rising trend of natural disasters.

Implementing mitigations can effectively reduce the potential risk and maintain insurability, protecting lives and properties⁶⁴. These measures prevent or limit damage

⁶² The irreversibility feature of climate change is further analyzed in Heal and Kristrom (2002).

⁶³ IPCC (2007) and Botzen (2009)

⁶⁴ According to IFRC (2001), worldwide investments of \$40 billions in disaster preparedness, prevention, and mitigation have reduced global economic losses for \$280 billion during the 1990s. Kreibich et al. (2005) analyzed the impact of building precautionary measures for the Elbe flood of Germany in 2002. They found that use of buildings and interior fitting adapted to flooding reduced damage to building by 46% and 53%, and damage to contents by 48% and 53%, respectively. Moreover, Kunreuther et al. (2009) model hurricane damage in New York, Texas, South Carolina, and Florida in situations with and without mitigations according to recent building code standards, The results for a 100-year hurricane indicate that mitigation could reduce potential losses by 61% in Florida, 44% in South Carolina, 39% in New York, and

against future natural disasters. For example, houses can be retrofitted or reinforced to withstand hurricanes, rising floods, or storm surges. However, these measures can have high up-front costs and the probability of a catastrophe seems to be remote and is often underestimated by many house owners. Investment on mitigations will only be worthwhile if the cost, which incurred in the short term, is less than the net expected benefits, which accrue over the long term. Thus, mitigations are more likely to be worthwhile when there is a risk in the near future, when people are concerned more about the future, or when the returns continue over longer terms. Due to uncertainty in the timing and magnitude of impacts, and difficulties in quantifying projected benefits and costs, it is often difficult to decide whether one should implement a specific adaptation to prepare for climate change. These uncertainties are incorporated into our model to justify the result of the benefit-cost analysis. The discount rate also matters when one would like to assess the value of implementing adaptations because it reflects people's attitudes toward the future and the degree of risk aversion. Individuals who have higher risk aversion or are more concern about the future tend to set a lower discount rate. As indicated by Kunreuther et al. (2009), extensive experimental evidence has shown that 'hyperbolic' temporal discounting has been applied by humans, meaning that even the next year has a very high discount rate. The implication lies in that homeowners tend to undervalue a mitigation investment, which requires high upfront cost but delayed expected benefits over time.

Mitigation measures can reduce the loss distribution and maintain insurability in the long term. Insurance can also effectively share and transfer the risk among a risk pool and

34% in Texas. Saving in Florida alone due to mitigation would result in \$51 billion for a 100-year and \$83 billion for a 500-year event.

thus help individuals to manage the residual risk that can not be eliminated by installing mitigations. With insurance, individuals can reduce financial exposures to catastrophes by speeding the recovery, maintaining business continuity, and reducing individual suffering. For example, Luechinger and Raschky (2009) estimated the utility losses caused by flood disasters in 16 European countries between 1973 and 1998 and empirically proved that insurance indeed enhance people's welfare. Based on this study, the presence of flood insurance almost fully mitigates the decline in the levels of life satisfaction resulting from flood disasters. In order to encourage mitigations, setting premiums at a level that reflects the underlying risk is very important. This principle allows insurers to provide lower premiums to homeowners who implement mitigations in their properties. In return, insurers can reduce the frequency and severity of claims. While some researches support the annually renewal contract⁶⁵, long term insurance has been proposed to be a way to stabilize insurance premiums for house owners in coastal areas. With potential climate change, it is vital to explore this issue more in depth and see how the price of multi-year policies will be affected by climate change and its associated uncertainties.

Scientific evidence shows that weather patterns are changing over the last century, causing sea level rise and intensifying the frequency and severity of future catastrophes⁶⁶.

⁶⁵ Vellinga et al. (2001) and Bouwer and Vellinga (2005) contend that allowing insurers to sign short term contracts and to adjust premiums and coverage over time ensures the solvency of the insurers against impacts of climate change

⁶⁶ Stern Review (2006) and IPCC (2007) provide more details on the scientific evidence of climate change and its impacts. According to IPCC (2007), global temperatures have increased approximately 0.76°C and sea level has risen about 20 centimeters since 1900. In addition, heat waves, heavy precipitations, and intensified tropical cyclones have emerged more frequently during late 20th century worldwide. Emanuel (2005), Webster et al. (2005), and Hoyos et al. (2006) suggest that an increased intensity and frequency of hurricanes are more likely to be induced by climate change through rising sea surface temperatures. Saunders and Lea (2008) further specify that 0.5°C increase in sea temperature is associated with 40% increase in hurricane frequency and activity.

The consequences for many coastal areas could be devastating. Homeowners in these areas can purchase property insurance to prepare for and recover from natural disasters. However, the availability and affordability of insurance will be diminished in the presence of climate change. Mitigation measures help to maintain the availability of affordable insurance for coastal properties, and thus are essential to provide better protection against losses in a long time scale. Due to myopia, underestimation of the risk, failure to learn from experience, interdependencies and budget constraints, homeowners are usually hesitate to pay the high cost of adaptations and receive the relative lower premium discount that reflect the reduction in expected annual losses⁶⁷. If adaptations are not installed, insurance price will keep rising and become less available in the next decades. Policymakers must consider these problems and incentivize homeowners to implement mitigations. These measures not only reduce the levels of potential damages but also help to maintain insurability in the long term. As pointed out in Mill (2007), insurers can regard climate change as a threat as well as a new business opportunity by developing innovative insurance products to stimulate adaptations. Confronting the rising trend of catastrophic risks associated with potential climate change, public and private sectors are suggested to cooperate to cope with the problem.

This study intends to analyze the implications of climate change for catastrophic risk and to examine the appropriateness of longer term insurance contracts to protect insurers against future catastrophic losses and changes in risk estimates over time due to climate change. Specifically, three major research questions have been raised and possible answers to them would be proposed in this study.

⁶⁷ Behavioral reasons why homeowners tend to delay or deny to install adaptation measures are explained in section 12.5 in Kunreuther and Michel-Kerjan (2009)

- Insurance markets functions well when loss distributions have certain statistical properties; well defined, low correlation both cross-sectional and inter-temporal, with thinner tail, etc. Are these properties preserved in a regime of climate change? How can we model the evolutions of losses under climate change?
- Weather losses can be mitigated by actions of the policyholder; however, the private returns from many mitigation investments may not be sufficient to warrant such investments on a short time scale. Climate change threatens to increase the level of risk and possibly also the returns from mitigation. How do we choose the optimal mitigation under conditions of climate change on a longer time scale?
- Given that climate change can impact both the demand for, and supply of, insurance, we may need to re-think the design of policies. In particular, it has been suggested that long term contracts might be appropriate both for the ability to facilitate mitigation and because they provide risk protection against future insurance availability and premium volatility. What is the difference between short and long term insurance contracts under climate change? What is the likely impact of cost of capital and Bayesian-updated serial correlation on risk capital and insurance premiums for short and long term policies?

The rest of this chapter is organized as follows. In section 3.2, I will investigate the statistical properties of the loss distribution in the regime of climate change and examine whether insurance market still function well given this properties. In section 3.3 and section 3.4, benefits and costs of mitigation measures on catastrophic losses for short and long time scales will be analyzed both by simulations and by using loss data in St Lucia, respectively. In particular, benefit-cost analysis on simulated losses and empirical losses

in St Lucia will be conducted both in the presence and in the absence of climate change for different time scales and discount rates. In section 3.5, historical storm activity will be used to measure the statistics of climate change factor in the near future. I will fit these statistics into two models to estimate annual insurance premiums for a longer contract and then compare the insurance prices between these two models. Finally, cost of capital and Bayesian-updated serial correlation will be taken into account in section 3.6 to estimate annual premiums for short term and long term catastrophe insurance contracts. Section 3.7 concludes.

3.2 Statistical Properties in the Regime of Climate Change

The insurance market functions well in a stable world, where loss distributions are well-defined (predictable), with low cross-sectional and inter-temporal correlations, and with thinner tail. In the presence of climate change, will these statistical properties still be preserved? In this section, I will explore this issue by developing a simple catastrophic risk model with potential climate change and illustrate the evolution of risk in a representative property for different features of climate change. These features include a climate change factor, the average level of risk changes over time, and the associated uncertainties regarding the factor.

3.2.1 A Simple Catastrophic Risk Model with Potential Climate Change

As indicated in section 1, climate change could play a critical role in estimating catastrophic risk. In order to highlight the impact of climate change on the severity of the catastrophic loss rather than that on the likelihood of a catastrophe, the model I construct

is based on the settings where a catastrophe⁶⁸ occurs with an annual probability p for a total T years and climate change influences only the catastrophic losses but not the frequency of the catastrophe. Timing of climate change is set to be a discrete uniform distribution⁶⁹ during the T years. That is, for each year, climate change may occur with possibility $1/T$. The occurrence of a catastrophe and the occurrence of climate change are mutually independent. Without climate change effect, the loss resulting from a catastrophe is assumed to be a constant, L ⁷⁰. As climate change occurs in year τ ($1 \leq \tau \leq T$), the potential catastrophic loss increases gradually with an annual growth rate, “ a ”, until year T . The annual growth rate, or the climate change factor, “ a ”, can be directly associated with an external index, such as the rise in the sea level. The higher the sea level, the greater the loss caused by a flood should a storm with heavy rain strike the hazard-prone area. The impact of the uncertainty of climate change on the potential loss works as follows, if climate change occurs in year τ and a storm hits the area after climate change in year t ($t > \tau$), the loss caused by the storm becomes $(1+a)^{t-\tau}L$. In contrast, the loss remains L if climate change occurs later than a storm. The simulation is executed by three steps: First, assume that climate change occurs in year τ , τ could be $1, \dots$, or T . The potential loss caused by a catastrophe is then specified as $L(s)$, $s=1, \dots, T$, where $L(s)=L$ for $s \leq \tau$ and $L(s)=(1+a)^{s-\tau}L$ for $s > \tau$. Next, the occurrence of a catastrophe is randomly simulated with an annual probability p during the T years for n times. n denotes the number of simulations. Given that climate change occurs in year τ , if a catastrophe occurs

⁶⁸ This simplest model assumes that catastrophes are only likely to occur once each year.

⁶⁹ Based on the results of our simulation, if the climate change time follows exponential distribution with the parameter $1/T$, the tail of the simulated distribution is very close to the worst case scenario, where climate change occurs in the first year. In addition, the mean losses are greater if climate change time is an uniform distribution than if it is an exponential distribution for all scenarios. Thus, the setting of uniform climate change time is more practical and more close to the real scenario.

⁷⁰ This is a simplified model. In section 3, this assumption will be relaxed and use the real data/empirical distribution of losses to substitute L .

in a year s , a catastrophic loss $L(s)$ specified above is assigned; otherwise, no loss is incurred. Finally, τ is supposed to follow discrete uniform distribution $[1, T]$. The statistics of the simulated losses will be collected. The parameters of the benchmark case of this simulation are as follows: $a=0.05$, $p=0.01$, $L=1$, $T=20$, $n=10^5$.

3.2.2 Exact Distribution of the Catastrophic Loss

In the case of no climate change, i.e., $a=0$, assuming other parameters are the same in the benchmark case, the aggregate loss is simply the sum of the annual loss for 20 years.

$\tilde{L} = \sum_{i=1}^{20} L_i$, where $P(L_i = 1) = 0.01$, $P(L_i = 0) = 0.99$, $i = 1, \dots, 20$. It is easy to deduce that

the aggregate loss follows a Binomial distribution with parameters $n=20$, $p=0.01$. The probability density function of the aggregate loss is given below.

$$P(\tilde{L} = k) = \binom{20}{k} (0.01)^k (0.99)^{20-k}, \quad k = 0, 1, \dots, 20.$$

The statistics of the aggregate loss, such as the expected value, standard deviation, skewness, kurtosis, and VaR (Value at Risk) at different confidence levels of the aggregate loss can be expressed as follows.

$$E(\tilde{L}) = np = 0.2$$

$$s.d.(\tilde{L}) = \sqrt{np(1-p)} = 0.445$$

$$skewness(\tilde{L}) = \frac{1-2p}{\sqrt{np(1-p)}} = 2.2024$$

$$kurtosis(\tilde{L}) = 3 - \frac{6}{n} + \frac{1}{np(1-p)} = 7.7505$$

$$VaR_{0.95} = VaR_{0.975} = 1, \quad VaR_{0.99} = 2$$

In the benchmark case, where $a=0.05$, the aggregate loss can be separated into two components: the aggregate loss prior to the occurrence of climate change and the

aggregate loss following the occurrence of climate change, i.e., $\tilde{L} = \sum_{i=1}^{\tau} \tilde{L}_{i,1} + \sum_{j=\tau+1}^{20} \tilde{L}_{j,2}$

, where $P(\tilde{L}_{i,1} = 1) = 0.01$, $P(\tilde{L}_{i,1} = 0) = 0.99$, $i = 1, \dots, \tau$,

$P(\tilde{L}_{j,2} = (1.05)^{j-\tau}) = 0.01$, $P(\tilde{L}_{j,2} = 0) = 0.99$, $j = \tau + 1, \dots, 20$.

Given the time that climate change occurs, τ , the conditional aggregate loss prior to climate change follows a Binomial distribution with parameters $n=\tau$, $p=0.01$, i.e.,

$\sum_{i=1}^{\tau} \tilde{L}_{i,1} | \tau \sim \text{Bin}(\tau, 0.01)$; however, the conditional aggregate loss posterior to climate

change, $\sum_{i=\tau+1}^{20} \tilde{L}_{i,2} | \tau$, can not be identified as a well-known distribution due to the cumulative

effect of climate change on the potential losses. Nevertheless, given that the climate

change could occur uniformly during 20 years, the expected value of the aggregate loss

can be calculated. The general form of the expected aggregate loss with potential climate

change can be presented as (3.1). The detailed derivations are available in section 3.8.

$$E(\tilde{L}) = \frac{p \cdot (T+1)}{2} + \frac{p \cdot (1+a)}{a} \cdot \left[\frac{(1+a)^T - 1}{a \cdot T} - 1 \right] \quad (3.1)$$

In the benchmark case, where $a=0.05$, $p=0.01$, $L=1$, $T=20$, the expected value of the aggregate loss is equal to 0.2422. Due to the complication of the aggregate loss, higher moments are difficult to be attained. The moment generating function of the aggregate loss with climate change is given as (3.11) in section 3.8. Although higher central moment of the distribution can generally be obtained by several times differentiating the log of the moment generating function of the aggregate loss with respect to the parameter θ and setting the value with $\theta=0$, the derivation in this case involves differentiation of a product with up to T terms and complicates the procedures. The derivation of the variance of the loss distribution by double expectation theorem is also available in section 3.8. The variance can be expressed in (3.18), but it is not an explicit formula since the term $E(\tilde{L}_2^2 | \tau)$ can not be derived. Thus, we have to resort to simulations to explore the statistical properties of the aggregate loss in the presence of climate change.

3.2.3 The Impact of Climate Change on Catastrophic Losses with a Certain Climate Change Factor

In this subsection, I will apply the catastrophic risk model constructed in section 2.1 to quantify the impact of climate change on catastrophic losses by setting a certain value to the climate change factor, “ a ”. Table 3.1 reports the statistics of the simulated losses for the case of no climate change factor ($a=0$) and the benchmark case ($a=0.05$) to compare the effect of climate change. These statistics include the expected value, the standard deviation, the skewness, the kurtosis of the losses, the VaR (Value at Risk), and the ES (Expected Shortfall). For VaR and ES, the values with confidence level of 95%, 97.5%, and 99% are presented. VaR and ES are indicators of the tail of loss distributions. In comparison of two distinct distributions, the greater the value of VaR or ES for the same

confidence level, the fatter tail the loss distribution. Value at Risk with confidence level α (VaR_α) simply denotes the α -percentile of the loss distribution. The proportion of the loss greater than VaR_α is at most $(1-\alpha)$. The formal definition of VaR is shown in (3.2).

$$VaR_\alpha = \inf\{l: P(L > l) \leq 1 - \alpha\} \quad (3.2)$$

Expected Shortfall (ES_α) is defined as (3.3), which is the expected value of the tail of the loss distribution with the loss threshold VaR_α .

$$ES_\alpha = E(L | L \geq VaR_\alpha) \quad (3.3)$$

The advantage of ES over VaR lies in that it is a coherent risk measure. The axiom of coherence was proposed by Artzner et al. (1997, 1999). VaR violates the axiom of subadditivity, thus is not a coherent risk measure. In the case of no climate change, the statistics from simulations are quite close to the corresponding values derived from the exact loss distribution. The expected loss from simulations is 0.2018 while the expected loss from exact distribution is 0.2. For the exact distribution of the aggregate loss with climate change, almost all statistics, except for the expected value, are not available to be compared with. The exceedance probability (EP) with a specific threshold is bounded by the Chernoff bounds⁷¹ which are based on the probability theory.

As shown in Table 3.1, in the presence of climate change (“a” increases from 0 to 0.05), the simulated expected loss increases only 20.32% while the tail statistics, such as

⁷¹ These bounds give exponentially decreasing bounds on tail distributions of the sum of independent random variables. It is better than the first or second moment based tail bounds such as Markov's inequality or Chebyshev inequality, which only yield power-law bounds on tail decay. The Chernoff bound of the distribution of the aggregate loss with climate change is presented in section 3.9. The lowest Chernoff bound is obtained by choosing the optimal parameter θ such that the bound is minimized. Figure A-1 in section 3.9 shows the plot of Chernoff bounds versus various thresholds for different time horizons.

VaRs and ESs, increase at least 26.60%. (The only exception is $\text{VaR}_{99\%}$, which increases only 16.57%. This may be due to too few samples in the extreme tail.) This result means that the climate change factor generally has more impact on the tail of the loss distribution rather than on the expected loss.

Figure 3.1 depicts the EPs for different climate change factors ($a=1, 0.05$, and 0.1). It indicates that climate change and its associated uncertainties are critical in modeling catastrophic risks and cannot be ignored, especially when the tail of the loss distribution is the focus of insurers or reinsurers. Climate change brings in higher inter-temporal correlations, which in turn lead to a fatter-tail loss distribution over time. The impact of climate change on the loss cannot be measured accurately, leading to unstable loss distributions, which create challenges to estimate the loss distribution with reliable parameters. These statistical properties could increase risk capital required and depress the supply or even the availability of insurance.

3.2.4 Uncertainty of Climate Change Factor

In section 3.2.3, the impact of climate change is certain (with a constant annual growth rate “ a ”). However, the impact of uncertain climate change should be of more concern for the insurance industry because the uncertainty is directly associated with the pricing and risk management of insurance contracts and insurance-linked instruments. This section further introduces the uncertainty of climate change factor into the simple catastrophic risk model. In particular, climate change factor, “ a ”, follows a discrete uniform distribution.

Table 3.2 reports the statistics of simulated losses for different settings on the uncertainty of the climate change factor. Column (1) is the case of a certain climate change, where “a”=0.05. Column (2) shows the statistics in the case of an uncertain climate change, where “a” takes three possible values: “a”=0.025, 0.05, or 0.075, each with probability of 1/3. Column (3) exhibits the statistics in the case of a more uncertain climate change, where “a” takes five possible values: “a”=0, 0.025, 0.05, 0.075, or 0.10, each with probability 1/5. Column (4) ((5)) shows the impact of the uncertainty on the statistics in term of percentage: percentage changes from column (1) to column (2) ((3)). Among all statistics, the tail probabilities are affected by the uncertainty of climate change factor to the greatest extent. As can be seen in Table 3.2, the contribution of uncertainty of climate change on the statistics of simulated losses most ranges from 0.62% to 4.62%. Nevertheless, the tail probabilities with threshold greater than the value of the house will increase substantially to over 9%.

Figure 3.2 exhibits the EP curves for different settings on the uncertainty of the climate change factor. The EP curve of “a=0.025~0.075” is quite close to the EP curve of “a=0.05”. The EP curve of “a=0.00~0.10” demonstrates a slightly fatter tail than the EP curve of “a=0.05”. However, a certain change of “a”, from 0.05 to either 0 or to 0.1, will cause a large shift of the EP curve, compared with the “uncertain” change (a=0.025~0.075 or 0~0.1). As demonstrated in Figure 3.3, the EP curve of “a=0.00~0.10” matches well with the EP curve of “a=0.06”. That is to say, the impact of the uncertainty (from “a=0.05” to “a=0~0.1”) is equivalent to the impact of a small certain change (from “a=0.05” to “a=0.06”). To sum up, the uncertainty of climate change factor leads to a

fatter-tail loss distribution, but its impact is small but positive, equivalent to a small certain increase in climate change factor.

3.3 Climate Change, Optimal Mitigations, and Time Scales

Catastrophic losses can be mitigated by investment on mitigation measures, which reduce both the expected value and the tail of the loss. The benefit comes from the reduction of potential losses while the cost is reflected in the price of the mitigation measures. For a short time scale, policyholders may hesitate to make the investment since short term benefit can not cover the cost. However, the threat of potential climate change stimulates us to consider risk from a longer term perspective as well as enhances the returns of mitigations. In this section, the optimal mitigation under conditions of climate change on a longer time scale will be explored.

3.3.1 Model Setting for Benefit-Cost Analysis

Without relevant scientific and engineering data and professional knowledge⁷², an empirical evaluation of mitigation costs and benefits is a daunting task. In this section, we conduct benefit-cost analysis by simulations rather than by empirical evaluations. The mitigation level is defined as the reduction of losses when a catastrophe destroys a property. For example, if a hurricane hits a house with a reinforced roof, the damage to the house will reduce 10% compared to the damage to the same house with no reinforced roof, the mitigation level is set to be 0.1. Mitigation cost should be an increasing and

⁷² With these data at hand, the most extensively used approach to determine the appropriate mitigation measure is by applying benefit-cost analysis. Smyth et al. (2009) employ benefit-cost analysis on various seismic retrofitting measures to mitigate losses arise from earthquakes in Istanbul area.

convex function of the mitigation level because homeowners have to pay higher price for a higher level of protection and the marginal cost of mitigations should increase with mitigation levels. In particular, mitigation cost is set to be $0.05 \cdot m + 0.1 \cdot m^2 + 0.05 \cdot m^3$, where m stands for the mitigation level. Once a catastrophe strikes an area, it will give rise to catastrophic cost, which includes the direct cost and the indirect cost. The direct cost is simply the expected loss caused by the catastrophe. The indirect cost, such as suspension of normal operations, interruption of public transportation, and costs of human lives, is linked to the tail of the catastrophic loss. When many homes are mitigated there is an added benefit to the insurer in the form of lower catastrophic losses and lower capital costs in addition to the reduction in claims from each individual policy. This additional benefit will be considered the reduction of the indirect cost. Suppose the indirect cost is proportional to tail probabilities and is set to be the weighted sum of the EPs with different thresholds. Specifically, the cost of catastrophe = $E[L]$ + Tail factor (L).

$$\text{Tail factor (L)} = \sum_{i=1}^9 \text{weight}(i) \cdot P(L > \bar{L}_i) \quad (3.4)$$

, $\text{weight}(i) = (0.5) \cdot (1 - (0.1) \cdot (i - 1))$, $i=1, \dots, 9$, and $\bar{L}_i = 1 + 0.5 \cdot (i - 1)$.

This setting allows the tail factor to assign more weights on extreme losses rather than moderate losses. For example, the weight on $P(L > 5)$ is 2.7 while the weight on $P(1.5 > L > 1.0)$ is only 0.5. The rationale of assigning more weight on a more extreme loss lies in that insurers will encounter a great deal of claims after an extreme disaster, causing them more likely to default on claims or even declare bankruptcy. In order to avoid the consequences of a larger loss, insurers would reserve more capital in advance.

The more capital reserved, the higher the cost of capital, thus the more weight to place in the extreme loss.

3.3.2 Issues on Discount Rates

The conclusions of benefit-cost analysis on a specific project are quite sensitive to the discount rates, especially when the project involves long time scales. The most popular debate which drew great attentions among economists on the global warming policy is the Stern Review on the Economics of Climate change, published in 2006 by the U.K. government. The Stern Review urges that prompt and sharp actions should be undertaken to abate potential catastrophic damages caused by greenhouse gas emissions. The major controversy on the results of Stern Review stems from the choice of very low discount rate. A large collection of comments on the Stern Review focuses on the discount rate issues, including Gollier (2006), Nordhaus (2007), Weitzman (2007), Dasgupta (2007), Heal (2009), and Mendelsohn (2008). The determination of the proper discount rate is associated with the pure rate of time preference, the measure of relative risk aversion, and the consumption growth over time, as established in the Ramsey equation. Based on Ramsey F. (1928), the equilibrium of real return on capital is governed by (3.5).

$$r = \delta + \eta \cdot g \quad (3.5)$$

r denotes the interest rate; δ is the pure rate of time preference; η stands for the elasticity of the marginal utility of consumption, or a measure of relative risk aversion; g represents the consumption growth rate. The choice of the appropriate discount rate involves ethical judgments on preference for intergenerational utility, preference for equality within the whole society, and concern about the balance between economic development and

environmental protection. Our simulations do not make a subjective judgment and thus specify various possible discount rates for analyzing the cost and benefit with optimal mitigation for different time scales.

3.3.3 Benefit-Cost Analysis by Simulations

The optimal mitigation level is determined by two opposite forces: A higher mitigation level costs more, but it reduces the expected losses and tail probabilities by abating the damage caused by a catastrophe. Thus, the optimal mitigation level can be achieved by minimizing the total cost, the sum of the price of mitigation measures and the damage. Figure 3.4 demonstrates the simulation results in the case of 20-years and 10% discount rate. The optimal mitigation level is 25%.

In order to analyze the benefit and cost of mitigations, I define optimal mitigation cost, benefit from optimal mitigation, net benefit from the optimal mitigation, and relative net benefit (RNB) from optimal mitigation as follows. Optimal mitigation cost is simply the price of mitigation measures at the optimal level of mitigation. Benefit from optimal mitigation reflects the reduction of losses resulting from the optimal mitigation. Net benefit from the optimal mitigation is the difference between benefit from optimal mitigation and optimal mitigation cost. Relative net benefit (RNB) from optimal mitigation is the ratio of net benefit from optimal mitigation to the total cost of no mitigation. The purpose to derive this quantity is to compare the relative magnitude of net benefit from mitigations among different time scales. Table 3.3 reports the summary of the cost/benefit of optimal mitigation for different time scales with different discount rates (0% and 5%). No matter which discount rate is specified, the optimal mitigation cost increases with time scales. For instance, if discount rate=0%, when the time scale

increases from 1 to 20, the optimal mitigation cost increase from 0 to 0.00563. This is because the longer time scale we are considering, the higher mitigation level will optimize the welfare (benefits minus costs). Cost reduction from optimal mitigation also magnifies with the time scale and with a higher speed, causing the net benefit (and the RNB) from optimal mitigation increases with time scales. In the case of 0% discount rate, RNB monotonically increases with the time scale from 0% to 45.44%. Based on the analysis, for a longer time scale, net benefit for all parties involved in catastrophic risk is more enhanced by investing in the mitigation measures.

Moreover, by comparing the optimal mitigation levels and net benefits from the optimal mitigation with various discount rates, there are two findings. First, the minimal time horizon that makes the investment on mitigation worthwhile increases with the discount rate. When the discount rate is 0%, the minimal time horizon that generates a positive optimal mitigation level is 2 years. As the discount rate rises to 5%, the minimal time horizon increases to 6 years. Second, for a given time scale, RNB from optimal mitigation always declines with discount rate. For instance, given time scale of 10 years, as the discount rate rises from 0% to 5%, RNB decreases from 32.49% to 4.73%. These findings suggest that a higher discount rate impedes the incentive to invest on mitigation measures, consistent with the first argument on the relationship between equity and climate change in Heal (2009): “We are less future-oriented—the CDR (consumption discount rate) is higher—and so we place less value on stopping climate change.” In sum, as human beings care more about the future, no matter in terms of setting a low discount rate or a longer time scale in modeling catastrophic risk, investing more on mitigation facilities will boost their utilities.

Climate change and mitigations have opposite effect on the total cost in the analysis. Climate change will increase total cost while mitigations will reduce total cost. Which effect dominates the other and how large is the aggregate effect? In order to answer these questions, Table 3.4 reports the total cost in four scenarios: (1) no climate change, no mitigation (2) no climate change, with mitigation (3) climate change, no mitigation (4) climate change, with mitigation. Climate change effect measures the increase of total cost due to climate change and is reflected in the discrepancy between scenario (3) and scenario (1) (or scenario (4) and scenario (2)). Mitigation effect gauges the decline of total cost due to mitigation measures and can be captured by the difference between scenario (2) and scenario (1) (or between scenario (4) and scenario (3)). Aggregate effect combines both effects and is measured in terms of the difference between scenario (4) and scenario (1). Table 3.5 shows climate change effect, mitigation effect, and aggregate effect by calculating the percentage change described above.

As expected, the total cost magnifies with climate change factor but declines with mitigations. In Table 3.4, if discount rate=0%, $T=5$, total cost of no mitigation no climate change is 0.0729, which is less than 0.0733, total cost of no mitigation with climate change but greater than 0.0509, total cost of optimal mitigation no climate change. In some cases, especially those with a long time scale and low discount rate, mitigation effect dominates climate change effect. This shows that the implementation of mitigation measures plays a very important role to reduce catastrophic losses even in the presence of climate change. Based on the observation in Table 3.5, climate change effect is generally more significant for discount rate=5%, but aggregate effect is more substantial for discount rate=0%. For example, in the case of $T=15$, if 5% discount rate is specified,

climate change increases total cost for 13.71% while mitigations decrease total cost for 11.82%; however, if 0% discount rate is specified, climate change increases total cost for 6.17% while mitigations decrease total cost for 38.23%. In addition, mitigation effect dominates for a longer time scale. In the case of 0% discount rate, if we increase the time scale from 10 years to 20 years, mitigation effect rises from 32.48% to 45.43%. These results justify the fact that returns of mitigation measures will be sufficient to warrant such investment for longer time scales and for individuals who care more about the future.

3.4 Simulations on Catastrophic Risk using Empirical Loss Data:

Hurricane Risk in St. Lucia

World Bank recently issues a report, which is a work collaborated with IIASA, RMA (Risk Management Solutions), and Wharton Risk Center, aiming to analyze the impact of cost-effective mitigation measures on the reduction of losses caused by natural disasters. This report contains several case studies, which quantitatively estimate the potential losses and implement cost-benefit analysis on various mitigation measures across different areas, including hurricane risk in St Lucia, flood risk in Jakarta, earthquake risk in Istanbul, and flood risk in Uttar Pradesh. In this section, we focus on the case study in St Lucia. EPs, losses, mitigation costs and the impact of four different mitigation measures (no mitigation, roof upgrade, opening protection, and roof & opening mitigations) on losses for different building types in various areas are provided by RMS. Because wood frame buildings in Canaries are proven to be the most effective one from

mitigations, we use this case as an example. The empirical data are incorporated into our simple catastrophic risk model to simulate the situation where climate change could occur in the next 20 years. The impact of climate change and mitigation measures will also be explored for various time scales.

3.4.1 Impact of Climate Change and Mitigation Measures

The first step is using Monte Carlo Simulation and interpolation to randomly generate EP curve that matches data points of the loss in St. Lucia in the World Bank Report. Next, the EP curve is incorporated into our simple model described in section 3.2.1 to estimate the impact of climate change on hurricane risk. Figure 3.5-3.8 depict the EP curves for the wood frame building in Canaries in 20 years for four different mitigation measures and different climate change factors. A comparison of these four mitigation measures indicates that the impact of climate change declines with more or stronger mitigation measures. As can be observed, the discrepancy between “ $a=0.1$ ” and “ $a=0$ ” narrows down to a great extent in the presence of mitigation measures. In addition, the tail of the EP curve with the same climate change factor becomes significantly thinner with roof & opening mitigations (AB) (Figure 3.8) than with roof mitigation (A) (Figure 3.6) or opening mitigation (B) (Figure 3.7). These findings can also be confirmed by comparing the quantities in Table 3.6 and Table 3.7. Table 3.6 (corresponding to Figure 3.5) reports the statistics of simulated losses for three climate change factors ($a=0$, 0.05, and 0.1) and no mitigation measure in 20 years, whereas Table 3.7 (corresponding to Figure 3.6) exhibits the same statistics with roof reinforcement (A). All statistics are reduced due to the roof mitigation, especially the tail statistics. For example, in the case of “ $a=0.05$ ”, expected loss reduces from 1.01 to 0.74, and $\text{VaR}_{95\%}$ also declines from 2.18

to 1.71 if roof mitigation is installed. Tail probability with threshold 2 also decreases from 0.07 to 0.02.

What is the effect of these mitigation measures in the St. Lucia case for different time horizons? Figure 3.9 (3.10) shows the EP curves for the wood frame building in Canaries with different mitigation measures in the absence of climate change, “ $a=0$ ” (in the presence of climate change, “ $a=0.05$ ”) when time horizon is 10 years. Figure 3.11 (3.12) depicts similar EP curves but with $T=20$ years. By comparing these EP curves, there are three findings:

1. The relative performance among the four mitigation measures is as follows. Roof & opening mitigation (AB) outperforms opening mitigation (B), which in turn outperforms roof mitigation (A), which follows by no mitigation.
2. As anticipated, the longer time scale, the greater the risk exposure, thus the fatter tail the loss.
3. The mitigation measures are more effective in the presence of climate change than in the absence of climate change. This suggests that the awareness of climate change will stimulate the investment on mitigations.

3.4.2 Benefit-Cost Analysis on Mitigations

Based on the statistics and tail probabilities of simulated loss obtained by inputting losses of St. Lucia for four different mitigation measures for the wood frame building in Canaries in the absence of climate change or in the presence of climate change, benefit-cost analysis can be conducted. Consistent with the World Bank Report, benefit from mitigation is defined as the reduction in the expected loss with a specific mitigation

measure compared with the expected loss with no mitigation. Benefit-cost ratio (B/C Ratio) is simply the benefit from mitigation divided by the cost of mitigation. According to the World Bank Report, a standard wood frame building in Canaries of St. Lucia has the value of \$100,000. Roof upgrade (A) costs \$9,200, opening protection (B) costs \$6,720, and roof & opening mitigation (AB) costs \$15,920. With this information and our simulation results for different time horizons, B/C Ratios for different mitigation measures can be easily derived.

Table 3.8 and Table 3.9 summarize benefits, costs and the B/C ratios of three different mitigation measures and various time scales for the wood frame building in Canaries of St. Lucia in the absence of climate change and in the presence of climate change, respectively. Based on numerical results in these two tables, the findings regarding to time scales are listed in the following.

1. Benefit from mitigation rises with the time scale and benefit from roof & opening mitigation (AB) is greater than benefit from opening mitigation (B), which in turn is greater than benefit from roof mitigation (A). When “a”=0 and T=10 years, benefit from roof & opening mitigation is 0.2014, benefit from opening mitigation is 0.1284, and benefit from roof mitigation is 0.1071.
2. As anticipated, B/C Ratio also rises with the time scale.
3. B/C Ratio of opening mitigation (B) is greater than B/C Ratio of roof & opening mitigation (AB), which in turn is greater than benefit from roof mitigation (A).

The reason why B/C Ratio of opening mitigation (B) is with the highest value lies in that its cost, \$6,720, is relative low compared with it benefit (the reduction in

expected loss). In addition, B/C Ratios in the absence of climate change are consistent with the B/C Ratios in the World Bank Report.

4. The minimal time scale that makes the investment in mitigation worthwhile (the minimal time scale such that $B/C \text{ Ratio} > 1$) reduces one year for roof mitigation (A) and opening mitigation (B) if climate change is present while it stays the same for roof & opening mitigation (AB). Specifically, in the absence of climate change, roof mitigation (A) is worthwhile to be implemented for at least 9 years; opening mitigation (B) is worthwhile for at least 6 years; roof & opening mitigation (AB) is worthwhile for at least 8 years. In contrast, in the presence of climate change, the minimal time horizon for roof mitigation (A) is 8 years; for opening mitigation (B) is 5 years; for opening mitigation (AB) is still 8 years. This implies that the presence of climate change will increase the return from mitigations.

Impact of climate change on the B/C Ratios can be observed by comparing Figure 3.13 and Figure 3.14. These two figures show the change of B/C Ratios for different mitigation measures over different time scales in the absence of climate change and in the presence of climate change, respectively, when discount rate=0%. As one would expected, B/C Ratios increase with the time horizon in both scenarios. However, they grow faster in the presence of climate change than in the absence of climate change. This phenomenon is more significant for longer time scales. For the time scale=20 years, B/C Ratio is above 4.5 in the presence of climate change while it is slightly below 4 in the absence of climate change. This indicates that the potential climate change and a longer

time scale enhance the relative benefits of cost-effective mitigation measures and thus encourage human to invest in mitigations.

Figure 3.15 summarizes the aggregate effects of climate change and mitigation. The most upper line is the EP curve with no climate change and no mitigation while the other three lines are EP curves with climate change and different mitigation measures (A, B, and AB) when time horizon is 20 years and discount rate=0%. This figure shows that the mitigation effect dominates the climate change effect. The impact of climate change on the EPs can be completely offset by implementing mitigation measures. This phenomenon is robust for all cases, no matter which discount rate is specified and how long time scales we are considering.

3.4.3 Uncertainty of Climate Change Factor in the Case of St. Lucia

Section 3.2.4 explored the impact of uncertainty of climate change factor on the tail statistics and on the tail of the simulated loss in the simple model. Here, we conduct similar sensitivity analyses of uncertain climate change factors. In order to introducing uncertainty of climate change factor, “a” is set to be a discrete uniform distribution and has five possible values: 0, 0.025, 0.05, 0.075, and 0.1. For each value of “a”, it occurs with an equal probability, 1/5. Due to the fact that the expected value of these possible climate change factor is 0.05, the case of “a=0.05” is taken as the benchmark case.

Introducing uncertainty of climate change factor is anticipated to result in a fatter tail of the loss distribution since a greater value of climate change factor will have more significant impact on the tail of the loss.

The EP curves for the wood frame house in Canaries of St Lucia for $T=20$ years are presented in Figure 3.16. As expected, the uncertain climate change factor (“ $a=0\sim0.1$ ”) has fatter tail than the certain climate change factor (“ $a=0.05$ ”). However, the impact of the uncertainty is minor compared to the certain shift of the climate change factor. Based on the tail statistics and tail probabilities in Table 3.10, the uncertain climate change factor (“ $a=0\sim0.1$ ”) is equivalent to a slightly higher climate change factor “ $a=0.06$ ”. This empirical result is consistent with the simulating one in section 3.2.4. Moreover, Figure 3.17 shows the EP curves of certain (“ $a=0.05$ ”) and uncertain climate change factor (“ $a=0\sim0.1$ ”) for $T=10$ years and $T=20$ years. For $T=10$ years, EP curves of certain and uncertain climate change factor almost match with each other, whereas for $T=20$ year, EP curve with the uncertain climate change factor exhibits a slightly fatter tail than EP curve with the certain climate change factor. This indicates that uncertainty in climate change factor has more impact for a long time scale. For a shorter time horizon, such as less than 10 years, the effect of uncertainty could be negligible. If one is thinking of 5 or even 10 year guaranteed renewability insurance policies the impact of uncertainty on “ a ” will not play a key factor in setting premiums.

3.4.4 Sensitivity Analysis of Annual Premiums

In this section, we use data from the case of St. Lucia and the simple catastrophic risk model to estimate insurance premiums and conduct sensitivity analysis on the patterns of the premiums across time scales.

$$\text{Annual Premium} = ((1+\lambda)E(L_T) + k \sigma_{L,T})/T \quad (3.6)$$

Annual insurance premiums are estimated based on (3.6), where $E(L_T)$ is the expected loss for T years; $\sigma_{L,T}$ denotes the standard deviation of the loss for T years; k is the indicator of hard/soft market for insurance industry. In hard market ($k=0.7$), premiums are relatively more responsive to the volatility of losses than premiums in soft market ($k=0.4$); λ reflects cost of capital for insurers; and T is the term of the insurance contract. Since insurers aggregate a risk pool to allocate the risk among policyholders, the cost of capital of insurers should be related to the tail probability of losses. Specifically, we suppose $\lambda = m \cdot P(L > \bar{L}_T)$, where m denotes a factor that reflects the financial vulnerability of the insurer. If the insurer is more financially fragile and has to charge a higher premium loading avoid potential bankruptcy, m is set to be higher. Because risk exposure for a longer time horizon should be higher, causing a fatter tail of the loss distribution, a higher threshold should be set to the loss of longer time horizons. In this section, m is set to be either 1 (a financially sound insurer) or 5 (a financially fragile insurer). In addition, $\bar{L}_1 = 0.25, \bar{L}_5 = 0.5, \bar{L}_{10} = 0.75, \bar{L}_{15} = 1, \bar{L}_{20} = 1.25$. In the subsequent subsections, a series of sensitivity analysis will be conducted regarding the insurance premium. The parameters of the benchmark case are as follows: “a”=0.05, d=0 (discount rate=0), m=5, k=0.4, and no mitigation. A wood frame building in Canaries of St. Lucia is assumed to be worth \$100,000.

The loss distribution is assumed to be the same in each year. In the case of no climate change, no mitigation, and a fixed cost of capital (cost of capital is not associated with the tail probability), the expected loss is proportional to the corresponding time horizon while standard deviation of the loss distribution is proportional to the square root of the corresponding time horizon. As a result, the annual premiums are predicted to be

decreasing with time horizons since the standard deviation of the loss grows slower than the time horizon. This is the rationale for proposing long term insurance. However, based on the settings, cost of capital for different time horizons rises with tail probabilities. Climate change will have a greater impact on the tail probabilities for longer time horizons, which will push up annual premiums. Thus, it is anticipated to observe a slightly declining or even a rising pattern of annual premiums for higher climate change factors.

3.4.4.1 Adaptations

Figure 3.18 demonstrates the patterns of insurance premiums across time horizons for different adaptations (no mitigation, roof upgrade (A), opening protection (B), and roof & opening mitigation (AB)). As can be observed, with no adaptation, annual insurance premiums generally rise with time horizons. The rising trend of the premium will be compromised if adaptations are installed. In the case of roof & opening mitigations (AB), premiums turn to decline with time horizons. Moreover, the adaptations significantly reduces annual premium, especially for longer time horizons. When roof & opening mitigations are installed, annual premiums decrease more than half of the original price. For instance, the annual premium for $T=5$ becomes one-third of the original price (\$3,851 with adaptations compared to \$11,308 without adaptation), and it plummets to one-fourth of the original price for $T=20$ (from \$15,859 to \$3,748). These findings support the proposition that long term insurance should be coupled with adaptations to incentivized policyholders' willingness to invest in mitigation measures.

Based on the pricing formula, insurance premiums are determined by three factors that are related to loss distributions: Expected losses per year, standard deviations of the loss

distribution per year, and tail probabilities. Further analysis on the contribution of these factors to the pattern of annual premiums indicates that, standard deviation per year, as predicted, declines with time horizons; however, due to the impact of climate change, expected losses per year and tail probabilities increase with time horizons. The level of increase in the tail probabilities is substantially reduced with more mitigation measures, causing the patterns of annual premiums shift from rising to slightly declining with time horizons.

3.4.4.2 Climate Change Factors

Figure 3.19 exhibits the patterns of annual premiums across time horizons for different climate change factors. As expected, in the absence of climate change ($a=0$), premiums slightly decline with time horizons. In contrast, in the presence of climate change, premiums show a rising trend. The rising trend is more substantial for a higher climate change factor. For the highest climate change factor ($a=0.1$) in Figure 3.19, the annual premium more than doubles from \$12,096 for $T=5$ years to \$26,226 for $T=20$ years. For the same increase in time horizon, the premium slightly decreases from \$10,387 to \$10,152 with no climate change. Thus, the impact of climate change on annual premiums is more significant for longer time horizons. The same as predicted in the beginning of the section. Considering climate change makes insurers raise the insurance price, especially for longer time horizons. This finding also highlights the importance of diversifying risk to reduce the likelihood of catastrophic losses.

Further analysis has been conducted on the contribution of expected losses per year, standard deviations per year, and tail probabilities to the pattern of annual premiums, respectively. In the case of no climate change, expected losses are stable, standard

deviations decline with time horizons, and tail probabilities concavely rise with time horizon. Consequently, annual premiums show a slightly declining pattern. In contrast, in the presence of climate change, expected losses exhibit an increasing trend. Although the patterns of standard deviations and tail probabilities are similar to the case of no climate change, introducing climate change alleviates the decreasing pattern of standard deviations as well as exacerbates the rising pattern of tail probabilities (from concavity to a little bit convexity). All the combination of these effects substantially pushes up the insurance price, especially for longer terms.

3.4.4.3 Discount Rates, Hard/Soft Market, and Financial Vulnerability of Insurers

The sensitivity analysis of annual premiums with respect to discount rates, hard/soft market, and financial vulnerability of insurers can be conducted in the same manner. I will only summarize principal results as follows.

A higher discount rate applied by insurers implies that the insurers do not care about future losses, giving rise to a large reduction of annual premiums. This phenomenon is more pronounced for a longer time horizons. For $T=20$ years, the annual premium decreases significantly to less than one-third of the original price (from \$15,859 to \$4,857) when the discount rate increases from 0% to 5%.

Annual premiums should be higher in hard market than in soft market. This effect is more substantial for short time horizons than for high time horizons. For instance, the price climbs 25% for $T=5$ (from \$11,308 to \$14,108) but it only rises 15% for $T=20$ (from 15,859 to \$18,231). The rational is associated with standard deviation since

standard deviation decreases with the time horizon, which in turn reduce the impact of hard/soft market.

Financially sound insurers will charge much less premiums because the cost of capital and risk capital reserved are much less than financially vulnerable insurers, especially for longer time horizons. For $T=20$ years, the annual premium can be saved for almost half of the original price (from \$15,859 to \$8165). However, for $T=5$ years, policyholder only save 35% of the annual premium (from \$11,308 to \$7,428). Thus, the benefit of long term insurance is greater if more financially sound insurers are willing to sell long term contracts.

3.4.5 Impact of Adaptations on Sensitivities of Annual Insurance Premium

Since implementing adaptations can reduce annual insurance premiums for a great amount, we will further explore the change in patterns of premiums from the case of no adaptation to the case of with adaptations. Roof and opening mitigations (AB) in St. Lucia is taken as the representative case of adaptations because it has the greatest impact on annual premiums. Figure 3.20 (3.21) shows the pattern of annual premiums across time horizons when adaptations and climate change factor (financial vulnerability of insurers) change simultaneously. A comparison of these cases indicates these findings:

1. The reduction in premiums due to adaptations dominates the reduction in premiums due to other factors. This result indicates that adaptations are the key factor for insurers to offer low annual premiums.
2. With no adaptation, premiums across time horizons mostly show a rising pattern, except for the case of no climate change. Nevertheless, with adaptations, premiums

across time horizons generally exhibit a flat or a declining pattern unless a extreme high climate change factor is specified (“ $a=0.1$ ” in Figure 3.20). This implies that with adaptations, insurers are more willing to provide lower or flat premiums for longer term contracts.

3. With adaptations, the sensitivities of annual premiums across time horizons with respect to other parameters decrease substantially. The most obvious example is the sensitivity of premium with respect to the financial vulnerability of insurers in Figure 3.21. As adaptations are installed, the difference of annual premiums between a financially sound insurer and a financially fragile insurer is very small. Thus, the implementation of adaptations will reduce total risk exposures, thus causing insurance premium less responsive to other risk-related factors.

3.5 Calculating Insurance Premiums Using Estimated Losses from Historical Storm Activities for Hurricane Risks in St. Lucia

In this section, storm activity statistics⁷³ are available for us to estimate climate change factor, “ a ”. Alternatively, we can use lognormal loss model to fit the loss statistics derived by those storm activity statistics. The aim is to calculate insurance premiums for different timescales with and without mitigation.

⁷³ Centre for Climate Change Economics and Policy, London School of Economics and Political Science (LSE) provided the estimates for future hurricane-associated loss distributions of St. Lucia.

3.5.1 Data Descriptions⁷⁴

The storm activity statistics are estimated by developing scenarios of storm activity over 5-year time scales using a simple model and estimating the probability and level of storm activity rate based on historical storm activity rate. Figure 3.22 provides the basic concept of the simple model. The next 5-year rate is predicted based on the mean over the past 5 years and the historical time series of the number of named storms and Cat 3-5 storms. ‘Upper’, ‘Middle’, and ‘Lower’ simply reflect different estimates of percentiles. For example, ‘Upper’ is 95% percentile, ‘Middle’ is the Median, and ‘Lower’ is the 5% percentile. Figure 3.23 and Figure 3.24 presents the annual number of named storms and Cat 3-5 storms in the Atlantic basin from 1950 to 2005. Based on these historical data and the simple model, the statistics of variability in storm activity over successive 5-year period can be shows as in Table 3.11. This tells us, for example, that on average, there is a 12% increase in the number of Cat 3-5 storms between any two successive 5-year periods; however, 35% of the time we will see a decrease in activity of at least 12%, and 35% of the time we will see an increase in activity of at least 22%.

These activity rates are used to adjust the frequencies of individual events in the RMS Caribbean Hurricane model and are converted into expected 5-year losses and standard deviation of losses, as exhibited in Table 3.12. These quantities are losses for 5 years. By assuming a simple constant growth rate, the annual growth rate can be derived. The climate change factor, “a”, is equivalent to the annual growth rate we obtained here.

⁷⁴ The descriptions are based on “Note on calculation of 5-year growth rates in hurricane wind-related losses and extension to longer time periods” by Nicola Ranger 2009.

3.5.2 Models and Methods

With the estimated annual growth rate of losses induced by hurricane risk in St. Lucia and EPs that are provided by RMS, two broadly defined models are candidates for calculating insurance premiums: the potential growth model and the lognormal loss model. The simple model we constructed in section 3.2.1 is the form of the potential growth model with exponential growth rate. However, since EPs of the present-day losses are available, we simply use these EPs along with the growth rate (or the climate change factor), “a”, obtained in section 3.5.1. The lognormal loss model assumes that the conditional loss follows lognormal distribution when a hurricane occurs.

3.5.2.1 Potential Growth Model

Hazard rate of climate change occurrence is defined as: $P_{\tau}(t) = 1 - (1 - P(0))^t = \frac{P(\tau = t)}{P(\tau > t - 1)}$

After iterations and normalization, the occurrence of climate change is summarized in Table 3.13.

L_t are randomly generated by Monte Carlo simulation based on the EPs. The distribution of ‘a’ is transformed from the 5 climate scenarios and are summarized in Table 3.14. For example, -0.085 and -0.0178 are point estimates of $\text{VaR}_{5\%}$ and $\text{VaR}_{35\%}$. Because the middle percentile is 20%, we assign 20% to the value of -0.085. By the same token, the middle percentile of $\text{VaR}_{35\%}$ and $\text{VaR}_{50\%}$ is 42.5%, thus we assign 22.5% (=42.5%-20%) to the value of -0.0178.

Different growth patterns, potential growth model can be divided into the following three models.

(1) Step Model

$$\sum_{t=1}^{\tau} L_t + \sum_{t=\tau+1}^T L_t (1 + \tilde{a})$$

(2) Linear Model

$$\sum_{t=1}^{\tau} L_t + \sum_{t=\tau+1}^T L_t (1 + (t - \tau) \cdot a)$$

(3) Exponential Model

$$\sum_{t=1}^{\tau} L_t + \sum_{t=\tau+1}^T L_t (1 + a)^{t-\tau}$$

3.5.2.2. Lognormal Loss Model

In this model, if a hurricane occurs, losses are assumed to be governed by a lognormal distribution, i.e., $\sum_{t=1}^{\tau} L_t, L_t \sim \log normal(\mu, \sigma)$. Based on the EPs, $\text{prob}(L_t=0)=0.5$.

Transformed conditional EPs can be obtained from the original EP curve. Since the estimates of μ and σ of the lognormal distribution can be derived by fitting EPs or the expected losses and standard deviation of losses, this model is also separated into two approaches: fit EPs and fit expected losses and standard deviation of losses.

(1) Fit EPs

μ and σ of the lognormal distribution are estimated by conditional EPs.

$$L = e^{\mu + \sigma z} \sim \log normal(\mu, \sigma)$$

$$F_L(\bar{L}) = P(e^{\mu + \sigma z} \leq \bar{L}) = P\left(z \leq \frac{\ln(\bar{L}) - \mu}{\sigma}\right) = \Phi\left(\frac{\ln(\bar{L}) - \mu}{\sigma}\right)$$

$$\Rightarrow \sigma \cdot \Phi^{-1}(F_L(\bar{L})) + \mu = \ln(\bar{L})$$

$\Phi^{-1}(F_L(\bar{L}))$ and $\ln(\bar{L})$ can be derived from the EPs. By minimizing the sum of the distances between RHS and LHS, these two parameters are estimated to be

$$\mu = 8.1867, \sigma = 1.077$$

(2) Fit expected losses and standard deviation of losses

Data supply unconditional means and standard deviations. Let L be the unconditional loss, and l be the conditional loss. Given $\text{prob}(L=0)=0.5$, the transformation between L and l is summarized as follows. $E(L) = 0.5E(l)$, $\text{Var}(L) = 0.5\text{Var}(l) + 0.25(E(l))^2$. Then,

$$\mu \text{ and } \sigma \text{ can be estimated by } \sigma = \sqrt{\ln\left[1 + \frac{\text{Var}(l)}{(E(l))^2}\right]}, \mu = \ln(E(l)) - \frac{\sigma^2}{2}.$$

5-year catastrophic losses are simulated based on (i) EPs of present-day losses (ii) expected loss and standard deviation of present-day losses (iii) expected losses and standard deviations of losses of 5 potential climate scenarios. Annual insurance premiums are calculated by the formula, $\text{Annual Premium} = ((1 + \lambda) \cdot E(L_T) + k \cdot \sigma_{L_T}) / T$, where $\lambda = m \cdot P(L > 0.5)$.

3.5.3 Simulation Results

Table 3.15 reports statistics, tail probabilities, and annual premiums for 5-year losses in different models. As can be seen, different assumptions on potential growth models

with empirical losses do not have great impacts on annual premiums. The step model produces the lowest premium while the exponential model estimates the highest premium. If the size of loss is set to be lognormal-distributed when a catastrophe occurs, the lognormal loss model with potential climate scenarios predicts a higher insurance price than the potential growth models. However, as shown in Table 3.16, the lognormal loss model does not fit empirical data well. This phenomenon becomes more pronounced in the extreme tail of losses (Percentile greater than 99.9%). Another limitation of lognormal loss model lies in that we can estimate the premiums only when the statistics of losses for the time horizon are available. If we only have expected losses and standard deviation of losses for 5 years, insurance premiums for 20 years can not be calculated.

Table 3.17 and Table 3.18 exhibit statistics, tail probabilities, and annual premiums under four different scenarios in the potential growth model and in the lognormal loss model, respectively: BCS represents Best Case Scenario, MLS denotes Most Likely Scenario, WCS corresponds to Worst Case Scenario, and UCS is Uncertain Climate Scenarios. Since UCS calculates the expected losses by assigning a distribution to five potential climate scenarios, it contains the most information on the empirical data compared to other scenarios and thus is the best estimate based on the data at hand. Lognormal loss model captures standard deviations of losses for each climate scenario, which are not used by potential growth model. Thus, lognormal loss model will estimate a higher insurance price than potential growth model. This phenomenon will be highlighted in the presence of different scenarios. Even though insurance prices are very close to each other for both models under MLS (potential growth model predicts \$5,438 while lognormal loss model estimates \$5,427), lognormal loss model shows greater

discrepancy between BCS and WCS (a higher premium in WCS and a lower premium in BCS). Under BCS, the annual premium estimated by lognormal loss model is \$3,515, much lower than that predicted by the potential growth model, \$5,091; however, under WCS, the annual premium of lognormal loss model is projected to be \$8,975, much higher than that simulated using potential growth model, \$6,002. Thus, it is not surprising that, under UCS, lognormal loss model, which incorporates the variance of 'a', estimates a higher annual premium. It simply reflects more uncertainties that will be encountered by the insurer in the future.

The choice between lognormal loss model and potential growth model lies in the availability of data. It would be more appropriate to assume the loss follows a lognormal distribution if one has sufficient data. For example, we have five different climate scenarios (ranging from 5% to 95%) as well as average annual losses and standard deviations of losses. The potential growth model uses only average annual losses but not standard deviations of losses. In order to extract more information from the data, fitting loss statistics to lognormal loss model is a better way to conduct the analysis. It is also worthy to note that lognormal loss model does not fit empirical data well in the extreme tail of losses, especially for percentiles greater than 99.9%. This is a disadvantage of all parametric distributions with few parameters. However, if one uses a generalized parametric distribution with more parameters, he will have higher variances of parameters, which could lead to imprecise estimates and higher type II errors.

3.6 Impact on Insurance Premiums in the Presence of Correlated Catastrophic Losses and Cost of Capital

In the previous section, we calculate the insurance premiums based on (3.6) by assuming that catastrophic losses are independent and that cost of capital is simply proportional to standard deviation of losses divided by the time horizon. In this section, the assumption on independence of losses over time will be relaxed and cost of capital will be reexamined based on Modigliani-Miller theorem. In addition, the expected loss for the next periods could adjust upward or downward based on the difference between the prior anticipated loss and the realized loss. This adjustment leads the losses to be Bayesian updated serial correlated. These impacts on insurance premiums will be illustrated quantitatively. Furthermore, the concept of risk capital proposed by Merton and Perold (1993) will be introduced to compare risk capitals required in two one-period contracts and a two-period contract.

3.6.1 Losses Are Correlated over Time

Assume L_t follows a T-variate unspecified distribution with parameters (μ, σ, ρ) . Annual premiums will be associated with the variance of the aggregate loss. Based on the derivation of section 3.10, $Var\left(\sum_{t=1}^T L_t\right) = T \cdot (1 + \rho(T-1)) \cdot \sigma^2$. The pattern of annual premium across time scales will depend on the correlation coefficient, ρ . Thus, except for the case of perfect correlation ($\rho=1$), annual premium will be decreasing across time scales. If L_t is assumed to be autoregressive (AR(1)) and stationary, i.e., $L_t = c + \phi L_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma^2)$, $|\phi| < 1$, the variance of the aggregate loss will be

$$Var\left(\sum_{i=1}^T L_i\right) = \frac{\sigma^2}{1-\varphi} \cdot \left[T + 2 \cdot \sum_{i=1}^{T-1} (T-i)\varphi^i\right], \text{ as calculated in section 3.10. We have a similar}$$

result: Unless losses over time are perfect correlated ($\varphi=1$), annual premium will be decreasing across time scales. Moreover, if we assume losses are independent while they are actually positively correlated, annual premiums will be underestimated because the variance is underestimated.

3.6.2 Cost of Capital

3.6.2.1 Modigliani-Miller theorem

The Modigliani-Miller theorem, proposed by Franco Modigliani and Merton Miller, forms the basis for modern thinking on capital structure (how much capital is allocated between debt and equity), though it is generally viewed as a purely theoretical result since it assumes away many important factors in the capital structure decision. These other reasons include bankruptcy costs, agency costs, taxes, information asymmetry.

Assume a perfect capital market (no transaction or bankruptcy costs; perfect information); firms and individuals can borrow at the same interest rate; no taxes; and investment decisions are not affected by financing decisions. The value of a company is independent of its capital structure (how a firm is financed is irrelevant to its value).

The analysis can be extended to take the effect of taxes into account. Under a classical tax system, the tax deductibility of interest makes debt financing valuable; that is, the cost of capital decreases as the proportion of debt in the capital structure increases. The optimal structure, then, would be to have virtually no equity at all.

Furthermore, bankruptcy cost is allowed to exist. In this case, there is an advantage to financing with debt (namely, the tax benefit of debts) and that there is a cost of financing

with debt (the bankruptcy costs of debt). The marginal benefit declines as debt increases, while the marginal cost increases, so that a firm can optimize its value by choosing the optimal ratio of debt and equity to use for financing.

3.6.2.2 Estimating Cost of Capital

Catastrophe insurance premiums should also consider cost of capital to insurers. Dollar cost of capital to insurers in our catastrophe risk model can be measured by multiplying cost of capital and expected total capital. In the benchmark case, we propose that cost of capital=2%. Bankruptcy cost is reflected in the dollar cost of capital that is required to maintain credit rating for insurers.

The procedures to estimate expected total capital (K_T) at initial for issuing T-year catastrophe insurance are shown as follows:

Set $K_{\tau,T}$ as the total capital that is required in year τ to maintain its credit rating for issuing T-year catastrophe insurance.

$$K_{\tau,T} = \left(\frac{1}{1+r} \right)^{\tau-1} \cdot \left(VaR_{C\%} \left(\sum_{t=\tau}^T \frac{L_t}{(1+r)^{t-\tau+1}} \right) - E \left(\sum_{t=\tau}^T \frac{L_t}{(1+r)^{t-\tau+1}} \right) \right), C \text{ is the critical percentile}$$

of Value at Risk ($VaR_{C\%}$). For example, $C=99.9$ represents the case that the capital is sufficient for 1-in-1000 years event. r denotes the discount rate.

$$\text{or } K_{\tau,T} = \left(\frac{1}{1+r} \right)^{\tau-1} \cdot \left(n \cdot \sigma \left(\sum_{t=\tau}^T \frac{L_t}{(1+r)^{t-\tau+1}} \right) \right), n \text{ is the number of standard deviation such}$$

that $n\sigma(L)$ is equal to the distance between $VaR_{C\%}(L)$ and $E(L)$.

The expected total capital that is required at initial of the contract to maintain its credit rating for issuing T-year catastrophe insurance is denoted as K_T .

$$K_T = \frac{1}{T} \cdot \sum_{\tau=1}^T K_{\tau,T} = \frac{1}{T} \cdot \sum_{\tau=1}^T \left(\frac{1}{1+r} \right)^{\tau-1} \cdot \left(VaR_{C\%} \left(\sum_{t=\tau}^T \frac{L_t}{(1+r)^{t-\tau+1}} \right) - E \left(\sum_{t=\tau}^T \frac{L_t}{(1+r)^{t-\tau+1}} \right) \right)$$

$$\text{or } K_T = \frac{1}{T} \cdot \sum_{\tau=1}^T K_{\tau,T} = \frac{1}{T} \cdot \sum_{\tau=1}^T \left(\frac{1}{1+r} \right)^{\tau-1} \cdot \left(n \cdot \sigma \left(\sum_{t=\tau}^T \frac{L_t}{(1+r)^{t-\tau+1}} \right) \right)$$

Cost of capital is simply (2%)*K_T

3.6.2.3 Annual Premium for T-year Catastrophe Insurance

$$\text{Total premium} = \sum_{t=1}^T \frac{E(L_t) + X_1}{(1+r)^t} + (2\%) \cdot T \cdot K_T + X_2$$

, where X₁ is the administrative cost of marketing a policy, X₂ is the upfront cost to insurer of marketing a policy.

Inputting K_T derived in section 3.6.2.2, total premium

$$= \sum_{t=1}^T \frac{E(L_t) + X_1}{(1+r)^t} + (2\%) \cdot \sum_{\tau=1}^T \left(\frac{1}{1+r} \right)^{\tau-1} \cdot \left(VaR_{C\%} \left(\sum_{t=\tau}^T \frac{L_t}{(1+r)^{t-\tau+1}} \right) - E \left(\sum_{t=\tau}^T \frac{L_t}{(1+r)^{t-\tau+1}} \right) \right) + X_2$$

$$\text{or } \sum_{t=1}^T \frac{E(L_t) + X_1}{(1+r)^t} + (2\%) \cdot \sum_{\tau=1}^T \left(\frac{1}{1+r} \right)^{\tau-1} \cdot \left(n \cdot \sigma \left(\sum_{t=\tau}^T \frac{L_t}{(1+r)^{t-\tau+1}} \right) \right) + X_2$$

Annual premium is simply the total premium divided by T.

Thus, annual premium

$$= \frac{1}{T} \cdot \left[\sum_{t=1}^T \frac{E(L_t) + X_1}{(1+r)^t} + (2\%) \cdot \sum_{\tau=1}^T \left(\frac{1}{1+r} \right)^{\tau-1} \cdot \left(VaR_{C\%} \left(\sum_{t=\tau}^T \frac{L_t}{(1+r)^{t-\tau+1}} \right) - E \left(\sum_{t=\tau}^T \frac{L_t}{(1+r)^{t-\tau+1}} \right) \right) + X_2 \right]$$

$$\text{or } \frac{1}{T} \cdot \left[\sum_{t=1}^T \frac{E(L_t) + X_1}{(1+r)^t} + (2\%) \cdot \sum_{\tau=1}^T \left(\frac{1}{1+r} \right)^{\tau-1} \cdot \left(n \cdot \sigma \left(\sum_{t=\tau}^T \frac{L_t}{(1+r)^{t-\tau+1}} \right) \right) + X_2 \right]$$

If we further set r=0, X₁=0, and X₂=0, the annual premium derived here can be compared with that estimated by (3.6'):

$$\text{Annual premium} = \frac{1}{T} \cdot \left[E\left(\sum_{t=1}^T L_t\right) + \underbrace{(2\%) \cdot n}_{\substack{\text{cost of capital} \\ \text{cost of capital}}} \cdot \underbrace{\sum_{\tau=1}^T \left(\sigma\left(\sum_{t=\tau}^T L_t\right) \right)}_{\substack{\text{sum of the standard deviation of the aggregate losses}}} \right] \quad (3.7)$$

, where $\sum_{\tau=1}^T \left(\sigma\left(\sum_{t=\tau}^T L_t\right) \right)$ stands for the sum of the standard deviation of the aggregate losses.

$$\text{Annual premium} = \frac{1}{T} \cdot \left[E\left(\sum_{t=1}^T L_t\right) + (0.4) \cdot \sigma\left(\sum_{t=1}^T L_t\right) \right] \quad (3.6')$$

$$\sum_{\tau=1}^T \left(\sigma\left(\sum_{t=\tau}^T L_t\right) \right) > \sigma\left(\sum_{t=1}^T L_t\right) = \text{standard deviation of aggregate losses}$$

Thus, using (3.6') to price insurance will underestimate annual premiums.

3.6.3 Annual Premiums with Correlations over Time and Cost of Capital

Taking climate change into account will introduce correlation of losses over time into the catastrophe model through the cumulative and uncertain effects of climate change.

In addition, the annual premium of catastrophe insurance would be increased when considering cost of capital by using (3.7) compared with using (3.6'). Table 3.19 (3.20) and Figure 3.25 (3.26) depict annual premiums of catastrophe insurance with no cost of capital and with cost of capital for a house with the value of \$1 million with no climate change (with climate change). As can be seen, the pattern of annual premiums changes because of cost of capital: with no cost of capital, annual premiums decline with the term of contracts; however, with cost of capital, annual premiums rise with the term of contracts. In addition, the presence of climate change increases the insurance premium. For example, with no climate change, if the term of the contract increases from 1 year to

5 years, the annual premium with no cost of capital declines from \$48,300 to \$44,500 while the annual premium with cost of capital rises to \$52,900. In contrast, in the presence of climate change, annual premiums for a 5-year contract are \$45,200 with no cost of capital and \$53,700 with cost of capital, respectively.

Table 3.19 and 3.20 also provide the sensitivity analysis on the cost of capital. The equivalent cost of capital shows the cost of capital such that a potential policyholder is indifferent between purchasing a multiple-year contract or sequential 1-period contracts. A cost of capital of 2% for the one-year contract is set as the benchmark. As can be seen, the cost of capital declines with the terms of contract indicating that a lower cost of capital is required to ensure that the annual premium will not increase for longer term contracts. For example, in Table 3.19, the equivalent cost of capital such that the annual premium is the same as an annual renewal contract decreases from 1.56% for a 2 year contract to 1.18% for a 5-year contract.

3.6.4 Bayesian-Updated Serial Correlation

Suppose that the realized catastrophic loss in the past year is underestimated compared with the anticipated loss one year ago, one would adjust the expected loss of the next year upward based on the realized loss. Through this yearly updated Bayesian process, the expected loss of the next year depends on both the long term trend of catastrophic risk and the realized losses of prior years. If it is the case, annual premiums could further rise due to the Bayesian-updated correlation because more capitals are required to be reserved for the uncertainty of updated expected losses in the subsequent periods.

The volatilities of no Bayesian-updated correlated process and Bayesian-updated correlated process in the presence and in the absence of growing trend of catastrophic losses have been derived in section 3.11. The results show that in either case, with Bayesian-updated correlation, the volatility will be higher. In addition, the annual premiums and capital reserves for insurers will also be greater compared with the case predicted by (3.6'), where Bayesian-updated correlations are considered. In particular, the annual premiums show a rising trend with respect to the time scale. As can be seen in the variance of Bayesian-updated process, the increment volatility of Bayesian-updated correlation ((3.25)-(3.24) in section 3.11),

$$\left\{ w^2 \cdot \left[\sum_{i=1}^{T-1} (1+a)^{2i} \cdot (T-i) + 2 \cdot \sum_{i=1}^{T-2} (1+a)^{2i} \cdot \frac{(T-i) \cdot (T-1-i)}{2} \right] + 2w \cdot \left[\sum_{\tau=1}^{T-1} (1+a)^i \cdot (T-i) \right] \right\} \cdot \sigma^2(\varepsilon)$$

, is positively proportional to the weight put on the realized loss in each period (w), the growth rate of catastrophic losses (a), and the time horizon (T). Thus, Insurers' updating their estimates of future losses based more weight on the revealed losses, potential climate change, and a longer term contract, will raise the annual premiums and the capital reserves that are required for insurers to support a given credit rating.

3.6.5 Comparison of Risk Capital in a 1-period Contract and a 2-period Contract

Based on Merton and Perold (1993), risk capital is defined as the smallest amount that can be invested to insure the value of the firm's net assets against a loss in value relative to the risk-free investment of those net assets. This concept of risk capital can be applied to the financing, capital budgeting, and risk management decisions of financial firms. In this study, we applied this concept to estimate risk capital that is required for insurers who wrote catastrophe insurance policies and to compare the amount of risk capital each period for two 1-period contracts and a 2-period contract. The major advantage of using

this concept of risk capital to estimate risk capital lies in that we can avoid determining the credit risk appropriate for comparing of insurance policies with different time scales. For insurers who issues catastrophe insurance, net assets are simply premiums collected minus the potential loss. Premiums are determined at the inception of the contract based on the expected loss if the price is actuarially fair, whereas the potential loss is set to follow a distribution with mean and variance. In this case, the risk capital can be expressed as the value of a European call option on the potential loss with exercise price equal to the premium.

3.6.5.1 Assumptions

1. L_1 and L_2 are prior belief about the loss distributions of period 1 and period 2. Specifically, they follow a normal distribution with parameters (μ_1, σ) and (μ_2, σ) , respectively.
2. There exists potential default risk for insurers when realized losses exceed premiums
3. The 2-period contract binds insurers to provide and policyholders to purchase catastrophe insurance in the beginning of period 2 at a pre-determined price
4. In the second 1-period contract, the price of catastrophe insurance can change based on the posterior belief about the loss distribution of period 2, which is affected by both the realized loss in period 1 and the prior belief about the losses
5. Posterior loss distribution of period 2 conditional on the realized loss of period 1, $L_2^N | L_1^R \sim N(\mu_2^N, \sigma)$, will be derived based on an updated process. If the realized loss of period 1 turns out to be greater (less) than the mean of the prior loss distribution of period 1, the mean of the posterior loss in period 2 will be adjusted

upward (downward) for the amount proportional to the discrepancy between the realized loss and the mean of the prior loss. More specifically,

$\mu_2^N = \mu_2 + w \cdot (L_1^R - \mu_1)$, $w > 0$. For simplicity, the updated process only changes the expected loss but not the standard deviation

6. Insurance premiums are actuarially fair and are determined prior to the realization of losses in each period. Thus, $P_1 = \mu_1$, $P_2 = \mu_2$, $P_2^N = \mu_2^N$
7. Risk capital is reflected in the price of the call option, which is available in capital markets for the insurer to hedge against the potential risk

3.6.5.2 Nature of Risk for Two 1-period Contracts and a 2-period Contract

In two 1-period catastrophe insurance policies, the premium of period 2 can be adjusted upward or downward based on the realized losses of period 1. The only risk is default risk when losses surpass premiums in each period. In contrast, in a 2-period catastrophe insurance policy, the premium is fixed in the beginning of period 1. The risk stems both from the catastrophe losses exceeding the premiums (default risk) and from the market-adjusted premiums exceeding the pre-determined premium (premium risk). The discrepancy of the nature of risk between two 1-period contracts and a 2-period contract leads to different amount of risk capital being reserved for different terms of insurance contracts.

3.6.5.3 Risk Capital for Two 1-period Contracts

In order to hedge the default risk, the insurer can purchase an option that payoff when the realized loss exceeds the premium they collected. When this situation occurs, the insurer will be indemnified for the difference between the realized loss and the premium

and avoid default on the claims. Risk capital to cover losses in period 1 is determined by

the value of the call option with payoff $(L_1 - P_1)^+$. $S_1 = E^Q[(L_1 - P_1)^+] = \frac{\sigma}{\sqrt{2\pi}}$, where

$E^Q[\cdot]$ represents the expected value under risk-neutral probability, whereas risk capital to cover losses in period 2 given the realized loss in period 1 is determined by the value of

the call option with payoff $(L_2^N | L_1^R - P_2)^+$. $S_2^N = E^Q[(L_2^N | L_1^R - P_2)^+] = \frac{\sigma}{\sqrt{2\pi}}$, where

$P_2 = \mu_2^N = \mu_2 + w \cdot (L_1^R - \mu_1)$. Please refer to section 3.12 for the derivations.

3.6.5.4 Risk Capital for a 2-period Contract

Assume the fixed premium predetermined at the beginning of period 1 is P_{LT} . The premium should be equal to the average of the expected losses in period 1 and period 2,

$$\text{i.e., } P_{LT} = \frac{\mu_1 + \mu_2}{2}.$$

If the realized loss of period 1 turns out to be greater than the prior expectation, market premiums in period 2 will adjust upward. Insurers who wrote a 2-period contract can not change the premium at period 2, creating higher risk for the existing policyholders.

Consequently, the insurers have to provide additional risk capital in period 1 to protect this possible incremental loss. The additional risk capital for insurers is equivalent to the option price with payoff $(P_2 - P_{LT})^+$, which option buyer pays to obtain the right to exercise the option if it turns out to be in-the-money ($P_2 > P_{LT}$). The option value can be derived as w units of call option with underlying L_1^R and strike price P_1 as follows.

$$E^Q[(P_2 - P_{LT})^+] = w \cdot E^Q[(L_1^R - P_1)^+] = w \cdot \frac{\sigma}{\sqrt{2\pi}}$$

In period 1, policyholders care about not only the default risk but also the premium risk, both of which will be triggered by the realized loss higher than anticipated in period

1. Risk capital to cover losses in period 1 is thus determined

$$\text{by } S_1^{LT} = S_1 + E^Q[(P_2 - P_{LT})^+] = (1 + w) \cdot \frac{\sigma}{\sqrt{2\pi}}.$$

If the realized loss does not exceed the anticipated loss, the premium in period 2 should be less than the fixed premium. In this case, risk capital to cover the loss in period 2 is determined solely by the value of the call option on default risk in period 2. The call

option has payoff $(L_2^N | L_1^R - P_2)^+$. $S_2^N = E^Q[(L_2^N | L_1^R - P_2)^+] = \frac{\sigma}{\sqrt{2\pi}}$, where

$$P_2 = \mu_2^N = \mu_2 + w \cdot (L_1^R - \mu_1)$$

3.6.5.5 Comparison of Risk Capital

A comparison of risk capital in period 1 for two 1-period contracts and a 2-period contract indicates that a 2-period contract requires more capital than a 1-period contract. The additional risk capital comes from the fact that policyholders in a 2-period contract face premium risk, which does not exist in a 1-period contract. In an annual-renewal contract, premiums can be adjusted based on the incoming realized losses in each period; however, in a long term contract, premiums are fixed in the inception of the contract and can not adjust upward even in the presence of rising trend of catastrophic losses. If the insurers purchase an option with payoff $(P_2 - P_{LT})^+$ in a 2-period contract, when the premiums in period 2 increases due to the high realized loss in period 1, insurers will be compensated for $(P_2 - P_{LT})$ for the option of hedging premium risk. Consequently, the insurers will hold $P_{LT} + (P_2 - P_{LT}) = P_2$ in the beginning of period 2. By purchasing this

option, insurers who wrote a 2-period contract will have the same payoff structure as those who wrote two 1-period contracts in the beginning the period 2, which makes 1-period and 2-period contracts comparable.

3.6.5.6 Example

An insurer provides periodically-renewal coverage for 2 periods:

$$L_1 \sim N(\mu_1, \sigma_1) \quad L_2 \sim N(\mu_2, \sigma_2) \quad \mu_1 = 1,000 \quad \mu_2 = 1,000 \quad \sigma_1 = \sigma_2 = 500$$

Assuming insurance is priced at an actuarially fair rate.

$$P_1 = 1,000 \quad P_2 = 1,000$$

$$\text{Risk capital in period 1} = (L_1 - P_1)^+ = 0.4 \sigma_1 = 200$$

$$\text{Risk capital in period 2} = (L_2 - P_2)^+ = 0.4 \sigma_2 = 200$$

Now suppose the insurer provides long term coverage for 2 periods and wants to determine risk capital in each period.

$$\text{Updated premium in period 2 is determined by } P'_2 = wL_1 + (1-w) \mu_2$$

Suppose $w=0.3$,

$$P'_2 = 0.3 L_1 + (0.7) 1,000 = 0.3L_1 + 700$$

Option for hedging premium risk of coverage in period 2

$$= (P'_2 - P_{LT})^+ = w(L_1 - P_1)^+ = (0.3) (0.4) \sigma_1 = 60$$

$$\text{Risk capital in period 2} = (L_2 - P_2)^+ = 0.4 \sigma_2 = 200$$

$$\text{Risk capital in period 1} = (L_1 - P_1)^+ + (P'_2 - P_{LT})^+ = 0.4 \sigma_1 + (0.3)(0.4) \sigma_1 = 200 + 60 = 260$$

The principal difference between risk capital of a 1-period contract and a 2-period contract lies in the option for hedging premium risk of coverage in period 2. This example explicitly illustrates the additional premium risk faced by policyholders of a 2-period contract compared to two 1-period contracts.

There exists offsetting effects for insurers to extend the term of contracts. On one hand, a longer term insurance contract encourages policyholders to invest on mitigations. Loss distributions will diminish to some extent after mitigations being implemented. On the other hand, a longer term contract increases risk capital required to be reserved in each period. In the above example, assume that a 2-period contract induces policyholders to implement mitigation measures, which reduce m proportion of the expected loss while an annual-renewal contract does not incentivize mitigation measures. The optimal level of mitigation such that 1-period and 2-period contracts are comparable can be determined by the proportion of expected loss being reduced, m^* . The risk capital in period 1 for a 2-period contract should be $260 \cdot (1 - m^*)$ while the risk capital in period 1 for a 1-period contract should be 200. Thus, $m^* = 23.08\%$. If a 2-period contract can induce mitigation such that the expected loss are reduced for more than 23.08%, insurers would like to promote 2-period contract rather than 1-period contract due to the less risk capital required. Based on the loss data for the wood-frame building in St. Lucia, expected loss will reduce at least 27.74% in the presence of mitigation measures. In this case, insurers are willing to issue a 2-period contract. In addition, there are other savings to the insurer from a 2-period contract—marketing costs and transaction costs associated with renewing the 1-year policy.

3.7 Conclusions

The impact of climate change on catastrophic risk has recently been the focus of researchers with a variety of backgrounds. In this study, I construct a simple catastrophic risk model with potential climate change, quantify the impact of potential climate change and associated uncertainties on catastrophic risk for longer timescales, conduct benefit-cost analysis of mitigation in reducing catastrophic losses with and without climate change, and estimate annual premiums for short-term and long-term catastrophe insurance under different scenarios. In particular, three research questions have been raised and potential answers to them have been proposed as follows.

First, will insurance market still function well given the distribution changes implied by climate change? In a regime of climate change, the catastrophic loss distributions are not well-defined since the impact of climate change on catastrophic losses is changing over time along with a great deal of uncertainty and ambiguity. Specifically, uncertainties with respect to the timing of climate change and the impact of climate change on potential losses have to be taken into account in the catastrophic risk model. In addition, climate change also leads to higher correlations and cumulative effect on inter-temporal losses, which in turn create a fatter-tail loss distribution. These climate-induced statistical properties could jeopardize the originally well-functioned insurance market, giving rise to higher premiums, increasing risk capital required, and depress the supply or even the availability of insurance.

Second, how does optimal mitigation change with climate change for a longer time scale? Benefit-cost analyses on the implementation of mitigation measures have been conducted both by simulations and by using hurricane losses in St Lucia, respectively. The simulation results indicate that the optimal mitigation level will rise with people's concern about the future (in terms of a lower discount rate or a longer time scale) while the empirical results suggest that homeowners will have more incentive to invest on mitigation in the presence of climate change and for longer time scales. Moreover, for the hurricane risk in St. Lucia, mitigation is empirically found to completely offset the impact of climate change.

Finally, what can we say about long term versus short term insurance contracts regarding the impact of cost of capital and Bayesian-updated serial correlations on risk capitals and premiums? Based on theoretical model I constructed, as cost of capital is taken into account, annual premiums increase with terms of the contract, whereas, after considering Bayesian-updated serial correlation on catastrophic losses, the volatility of aggregate losses rises compared to the case of no serial correlation. If the posterior expected loss will be adjusted based on the realized losses in the past periods, the risk capital in the sense of Merton and Perold (1993) has been proven to be greater for a 2-period contract than a 1-period contract due to the additional premium risk borne by the insurer. However, longer term contracts incentivize mitigation measures and reduce marketing and renewal transaction costs. The appropriate terms of insurance contract coupled with other risk transfer instruments can be determined by balancing the trade-off.

3.8 Future Research on Long Term Insurance

Potential climate change will create challenges in catastrophic risk management. The consequences can be limited if we implement effective mitigation measures and design appropriate risk sharing arrangements in advance. Mitigation measures have been shown to effectively reduce losses for all parties involved even with potential climate change. Longer term insurance contracts can incentivize mitigation measures⁷⁵ but will increase risk capital required and premiums due to the increased uncertainty regarding future losses for longer time scales. Future research would compare long term insurance contracts for various periods with an annual renewal contract with and without mitigation with different scenarios regarding climate change include the case of no climate change.

The case of no climate change will be deemed as the benchmark, the trade-off of long term contracts and annual renewal ones will involve the benefits of mitigation measures and the extra costs associated with holding reserve capital. A longer term insurance contract encourages policyholders to implement mitigation measures, but it raises risk capital and the cost of capital. How can one determine the appropriate terms of insurance contract to balance the trade-off between short term and long term contracts? Annual insurance premiums can be used as the criteria. Mitigation reduces the catastrophic loss and also the premium while higher cost of capital pushes up the premium. In order to conduct this analysis, we need empirical estimates of cost of capital by insurers as well as amount of risk capital reserved by insurers as a function of catastrophic losses and

⁷⁵ Based on Kunreuther, H. and Michel-Kerjan, E., 2009, *At War with the Weather: Managing Large-Scale Risks in a New Era of Catastrophes*, MIT Press

contract length; we also need to estimate the extent to which catastrophic losses can be reduced by different mitigation measures.

With potential climate change, similar trade-offs between a longer term contract and an annual renewal contract need to be analyzed. Climate change may reinforce both the benefit and the cost of longer term contracts by raising the returns from mitigation, and by increasing the amount of reserve capital. How will the reduction (increase) of annual premiums due to mitigations (cost of capital) change with potential climate change for different time scales? In addition to applying loss data to create EP curves with and without mitigation for the risk-prone regions, we will also estimate the impact of climate change on catastrophic losses (such as storm activities regarding hurricane risk or sea level rise regarding flood risk). This impact can be measured based on the catastrophic loss event table provided by Willis Re, which provides the event type, the rate of occurrence, the expected loss, the independent volatility, the correlated volatility, and risk exposures.

In this study, a simple two-period model was constructed to illustrate the additional risk capital required for a two-period contract compared with a one-period contract. I also calculated the minimal level of mitigation stimulated by a two-period contract such that two one-period contracts are comparable to a two-period contract in terms of risk capital. If we would like to apply this approach to evaluate the trade-off between mitigation measures and the length of contracts, more challenges will appear. First, it will be difficult to estimate catastrophic loss distributions with reliable parameters for each period, especially with potential climate change. Second, even if we can approximately estimate the loss distribution each period, it is unlikely that losses over time are

independent. With correlated losses, risk capital cannot be measured by the value of a call options with a simply form of payoff. For example, the payoff of a call option to hedge against premium risk in period one for a three-period model may depend on the correlated realized losses in the first two periods. More advanced option pricing techniques may be required to solve the problem. Third, if the losses over time are assumed to be independent and stable, the weights placed on the realized loss in the previous periods are unlikely to be constant. People tend to over-adjust large losses and under-adjust small losses. This behavior will further raise the risk capital for hedging against premium risk. In order to capture the effect of this behavior on the weights, an experimental study would be more suitable than from the empirical data.

Appendix

3.9 Statistics of Exact Losses with Potential Climate Change

The aggregate loss can be presented as two components, the aggregate loss prior to climate change and the aggregate loss posterior to climate change, i.e., $\tilde{L} = \tilde{L}_1 + \tilde{L}_2$.

\tilde{L}_1 denotes the aggregate loss prior to climate change and is the sum of annual losses

before the occurrence of climate change, i.e., $\tilde{L}_1 = \sum_{i=1}^{\tau} \tilde{L}_{i,1}$, where

$$\tilde{L}_{i,1} = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability } 1-p \end{cases} \quad \text{for } i = 1, \dots, \tau$$

\tilde{L}_2 represents the aggregate loss posterior to climate change and is the sum of annual

losses after the occurrence of climate change, i.e., $\tilde{L}_2 = \sum_{j=\tau+1}^T \tilde{L}_{j,2}$, where

$$\tilde{L}_{j,2} = \begin{cases} (1+a)^{j-\tau} & \text{with probability } p \\ 0 & \text{with probability } 1-p \end{cases} \text{ for } j = \tau+1, \dots, T$$

With the above settings, the conditional aggregate loss prior to climate change follows

Binominal distribution, i.e., $\sum_{i=1}^{\tau} \tilde{L}_{i,1} \mid \tau \sim \text{Bin}(\tau, p)$, whereas the conditional aggregate loss

after climate change can not be identified. Nonetheless, the conditional expected loss is derived as follows.

$$E\left(\sum_{j=\tau+1}^T \tilde{L}_{j,2} \mid \tau\right) = \sum_{j=\tau+1}^T E[\tilde{L}_{j,2} \mid \tau] = \sum_{j=\tau+1}^T p \cdot (1+a)^{j-\tau} = \frac{p(1+a)}{a} \cdot [(1+a)^{T-\tau} - 1]$$

$$\text{Thus, } E(\tilde{L}) = E\left(\sum_{i=1}^{\tau} \tilde{L}_{i,1} + \sum_{j=\tau+1}^{20} \tilde{L}_{j,2}\right) = E\left[\underbrace{E\left(\sum_{i=1}^{\tau} \tilde{L}_{i,1} \mid \tau\right)}_{=p} + \underbrace{E\left(\sum_{j=\tau+1}^T \tilde{L}_{j,2} \mid \tau\right)}_{=\frac{p(1+a)}{a}[(1+a)^{T-\tau}-1]}\right]$$

$$= p \cdot E(\tau) + \frac{p(1+a)}{a} \cdot [E((1+a)^{T-\tau}) - 1]$$

$$\left(\tau \sim \text{discrete } U(1, T) \therefore E(\tau) = \frac{T+1}{2}, \quad E((1+a)^{T-\tau}) = \sum_{\tau=1}^T (1+a)^{T-\tau} \cdot \frac{1}{T} = \frac{(1+a)^T - 1}{a \cdot T}\right)$$

$$= \frac{p \cdot (T+1)}{2} + \frac{p \cdot (1+a)}{a} \cdot \left[\frac{(1+a)^T - 1}{a \cdot T} - 1\right] \quad (3.8)$$

By LHospital Rule, as $a \rightarrow 0$, $\frac{p \cdot (1+a)}{a} \cdot \left[\frac{(1+a)^T - 1}{a \cdot T} - 1 \right] \rightarrow p \cdot \frac{T-1}{2}$. In equation (3.8),

the second term approaches $p \cdot \frac{T-1}{2}$ while the first term does not depend on a and

equates $p \cdot \frac{T+1}{2}$, thus the expected aggregate loss approaches $T \cdot p$, which verifies the

fact that if no climate change, $a=0$, $\tilde{L} \sim \text{Bin}(T, p) \Rightarrow E(\tilde{L}) = T \cdot p$.

Moreover, the conditional moment generating functions of the two components of the aggregate losses are given by these two equations:

$$M_{\tilde{L}_1|\tau}(\theta) = E(e^{\theta \cdot \tilde{L}_1} | \tau) = (pe^\theta + 1 - p)^\tau \quad (3.9)$$

$$M_{\tilde{L}_2|\tau}(\theta) = E(e^{\theta \cdot \tilde{L}_2} | \tau) = \prod_{k=1}^{T-\tau} (pe^{\theta(1+a)^k} + 1 - p) \quad (3.10)$$

Thus, the moment generating function of the aggregate loss can be written as follows:

$$\begin{aligned} M_{\tilde{L}}(\theta) &= E[e^{\theta \cdot \tilde{L}}] = E_\tau \left[E(e^{\theta \cdot \tilde{L}_1} | \tau) \cdot E(e^{\theta \cdot \tilde{L}_2} | \tau) \right] \\ &= \frac{1}{T} \cdot \sum_{\tau=1}^T \left[(pe^\theta + 1 - p)^\tau \cdot \prod_{k=1}^{T-\tau} (pe^{\theta(1+a)^k} + 1 - p) \right] \end{aligned} \quad (3.11)$$

We can verify that the expected loss can be derived by differentiating the log of the moment generating function of the aggregate loss with respect to the parameter θ at the value with $\theta=0$.

$$\begin{aligned}
E(\tilde{L}) &= \frac{\partial \log(M_{\tilde{L}}(\theta))}{\partial \theta} \Big|_{\theta=0} \\
&= \partial \log \left\{ \sum_{\tau=1}^T \left[(pe^{\theta} + 1 - p)^{\tau} \cdot \prod_{k=1}^{T-\tau} (pe^{\theta(1+a)^k} + 1 - p) \right] \right\} \Big|_{\theta=0} \quad (3.12) \\
&= \frac{1}{T} \sum_{\tau=1}^T \left(\tau \cdot p + p \cdot \sum_{k=1}^{T-\tau} (1+a)^k \right) = \frac{p \cdot (T+1)}{2} + \frac{p \cdot (1+a)}{a} \cdot \left[\frac{(1+a)^T - 1}{a \cdot T} - 1 \right]
\end{aligned}$$

Nevertheless, higher moments are much harder to be obtained compared with the expected value since the procedures involve differentiation of a product with up to T terms.

By Double Expectation Theorem, the variance of the aggregate loss can be expressed as this formula: $Var(\tilde{L}) = E[Var(\tilde{L} | \tau)] + Var[E(\tilde{L} | \tau)]$ (3.13)

$E(\tilde{L} | \tau)$ has been obtained in equation (3.8). Therefore, the second term in equation (3.13) can be derived as follows.

$$\begin{aligned}
Var[E(\tilde{L} | \tau)] &= Var\left(\tau p + \frac{p(1+a)}{a} \cdot [(1+a)^{T-\tau} - 1]\right) \\
&= p^2 Var(\tau) + \frac{p^2(1+a)^2}{a^2} \cdot Var[(1+a)^{T-\tau}] \\
&\left(\begin{aligned} &\tau \sim \text{discrete } U[1, T] \therefore Var(\tau) = \frac{T(T+1)(2T+1)}{6} - \frac{(T+1)^2}{4} \\ &Var[(1+a)^{T-\tau}] = E[(1+a)^{2(T-\tau)}] - (E[(1+a)^{T-\tau}])^2 = \frac{1}{T} \sum_{\tau=1}^T (1+a)^{2(T-\tau)} - \left(\frac{1}{T} \sum_{\tau=1}^T (1+a)^{T-\tau} \right)^2 \\ &= \frac{(1+a)^{2T-1} - 1}{T \cdot a} - \left(\frac{(1+a)^T - 1}{T \cdot a} \right)^2 \end{aligned} \right)
\end{aligned}$$

$$= p^2 \left[\frac{T(T+1)(2T+1)}{6} - \frac{(T+1)^2}{4} \right] + \frac{p^2(1+a)^2}{a^2} \cdot \left[\frac{(1+a)^{2T-1} - 1}{T \cdot a} - \left(\frac{(1+a)^T - 1}{T \cdot a} \right)^2 \right] \quad (3.14)$$

In order to derive the first term of equation (3.13), having the value of $Var(\tilde{L} | \tau)$ is the requirement. Since the independence between \tilde{L}_1 and \tilde{L}_2 given the timing of climate change τ , $Var(\tilde{L} | \tau) = Var(\tilde{L}_1 | \tau) + Var(\tilde{L}_2 | \tau)$. (3.15)

$Var(\tilde{L}_1 | \tau) = \tau p(1-p)$ can be easily deduced by discovering that $\tilde{L}_1 | \tau \sim Bin(\tau, p)$.

However, the functional form of the first term of equation (3.15) is difficult to obtain.

Since $E(\tilde{L}_2 | \tau)$ has been known in equation (3.8), we decompose the variance into two components: $Var(\tilde{L}_2 | \tau) = E(\tilde{L}_2^2 | \tau) - [E(\tilde{L}_2 | \tau)]^2$. (3.16)

, where $E(\tilde{L}_2 | \tau) = \frac{p(1+a)}{a \cdot T} [(1+a)^{T-\tau} - 1]$.

$$\begin{aligned} \text{Thus, } E[Var(\tilde{L} | \tau)] &= p(1-p)E(\tau) + E[E(\tilde{L}_2^2 | \tau)] - \frac{p^2(1+a)^2}{a^2 \cdot T^2} E[(1+a)^{T-\tau} - 1]^2 \\ &= p(1-p) \frac{T+1}{2} + E[E(\tilde{L}_2^2 | \tau)] - \frac{p^2(1+a)^2}{a^2 \cdot T^2} \left[\frac{(1+a)^{2T} - 1}{T((1+a)^2 - 1)} - 2 \frac{(1+a)^T - 1}{aT} + 1 \right] \end{aligned} \quad (3.17)$$

We have no clue to derive the second term of RHS in equation (3.17) and set it to be A.

In sum, variance of loss is given as follows.

$$Var(\tilde{L}) = E[Var(\tilde{L} | \tau)] + Var[E(\tilde{L} | \tau)]$$

$$\begin{aligned}
&= p(1-p)\frac{T+1}{2} + \underbrace{E\left[E\left(\tilde{L}_2^2 \mid \tau\right)\right]}_A - \frac{p^2(1+a)^2}{a^2 \cdot T^2} \left[\frac{(1+a)^{2T} - 1}{T((1+a)^2 - 1)} - 2 \frac{(1+a)^T - 1}{aT} + 1 \right] \\
&+ p^2 \left[\frac{T(T+1)(2T+1)}{6} - \frac{(T+1)^2}{4} \right] + \frac{p^2(1+a)^2}{a^2} \cdot \left[\frac{(1+a)^{2T-1} - 1}{T \cdot a} - \left(\frac{(1+a)^T - 1}{T \cdot a} \right)^2 \right] \quad (3.18)
\end{aligned}$$

3.10 Chernoff Bounds

$$\begin{aligned}
&\text{By section 3.8, } E\left[e^{\theta \cdot \tilde{L}}\right] = \frac{1}{T} \cdot \sum_{\tau=1}^T \left[\left(pe^{\theta} + 1 - p \right)^{\tau} \cdot \prod_{k=1}^{T-\tau} \left(pe^{\theta \cdot (1+a)^k} + 1 - p \right) \right] \\
P(\tilde{L} \geq x) &\leq \frac{E\left[e^{\theta \cdot \tilde{L}}\right]}{e^{\theta \cdot x}} = e^{-\theta \cdot x} \cdot \frac{1}{T} \cdot \sum_{\tau=1}^T \left[\left(pe^{\theta} + 1 - p \right)^{\tau} \cdot \prod_{j=1}^{T-\tau} \left(pe^{\theta(1+a)^j} + 1 - p \right) \right] \quad (3.19)
\end{aligned}$$

where $\theta > 0$

The Chernoff bound equates the minimal value of the RHS in equation (3.19), which are derived by finding the optimal positive θ . For example, the Chernoff bound (0.6207) of the exceedance probability with threshold of 1 in Table 3.2.1 is determined by setting $T=20$, $p=0.01$, $a=0.05$, and searching for the optimal $\theta=0.9911>0$.

3.11 Pattern of Annual Insurance Premium when Losses Are Correlated over Time

1. Assume L_t follows a T-variate unspecified distribution with parameters (μ, σ, ρ)

$$\begin{aligned}
Var\left(\sum_{t=1}^T L_t\right) &= Cov\left(\sum_{t_1=1}^T L_{t_1}, \sum_{t_2=1}^T L_{t_2}\right) = \sum_{t_1=1}^T \sum_{t_2=1}^T cov(L_{t_1}, L_{t_2}) \\
&= T \cdot Var(L_t) + \sum_{t_1 \neq t_2} cov(L_{t_1}, L_{t_2}) \\
&= T \cdot \sigma^2 + (T^2 - T) \cdot \rho \cdot \sigma^2 \\
&= T \cdot (1 + \rho(T-1)) \cdot \sigma^2 \tag{3.20}
\end{aligned}$$

$$\text{If } \rho=0, Var\left(\sum_{t=1}^T L_t\right) = T \cdot \sigma^2$$

$$\text{If } \rho=1, Var\left(\sum_{t=1}^T L_t\right) = T^2 \cdot \sigma^2 \Rightarrow \text{Annual premium will be constant}$$

$$\text{If } 0 < \rho < 1, Var\left(\sum_{t=1}^T L_t\right) = T \cdot (1 + \rho(T-1)) \cdot \sigma^2 \Rightarrow \text{Annual premium will be decreasing over}$$

time

2. L_t is autoregressive (AR(1)) and stationary

$$L_t = c + \phi L_{t-1} + \varepsilon_t, \varepsilon_t \sim N(0, \sigma^2), |\phi| < 1$$

$$E(L_t) = \mu = \frac{c}{1-\phi}, Var(L_t) = \frac{\sigma^2}{1-\phi}, Cov(L_{t+n}, L_t) = \frac{\sigma^2}{1-\phi} \cdot \phi^{|n|}$$

$$\begin{aligned}
Var\left(\sum_{t=1}^T L_t\right) &= Cov\left(\sum_{t_1=1}^T L_{t_1}, \sum_{t_2=1}^T L_{t_2}\right) = \sum_{t_1=1}^T \sum_{t_2=1}^T cov(L_{t_1}, L_{t_2}) \\
&= \frac{\sigma^2}{1-\varphi} \cdot \begin{bmatrix} 1 & \varphi & \dots & \varphi^{T-2} & \varphi^{T-1} \\ \varphi & 1 & \dots & \varphi^{T-3} & \varphi^{T-2} \\ \vdots & & \ddots & & \vdots \\ \varphi^{T-2} & \varphi^{T-3} & \dots & 1 & \varphi \\ \varphi^{T-1} & \varphi^{T-2} & \dots & \varphi & 1 \end{bmatrix} \\
&= \frac{\sigma^2}{1-\varphi} \cdot \left[T + 2 \cdot \sum_{i=1}^{T-1} (T-i)\varphi^i \right] \tag{3.21}
\end{aligned}$$

$$T + 2 \cdot \sum_{i=1}^{T-1} (T-i)\varphi^i < T^2 \Leftrightarrow \text{annual premium} \downarrow \text{ with } T$$

$$\text{If } \varphi=0, T + 2 \cdot \sum_{i=1}^{T-1} (T-i)\varphi^i = T$$

$$\text{If } \varphi=1, T + 2 \cdot \sum_{i=1}^{T-1} (T-i)\varphi^i = T^2 \Rightarrow \text{Annual premium will be constant}$$

$$\text{If } -1 < \varphi < 1, T + 2 \cdot \sum_{i=1}^{T-1} (T-i)\varphi^i < T^2 \Rightarrow \text{Annual premium will be decreasing over time}$$

3.12 The Comparison of Volatilities with no Bayesian-updated

Correlated Process and with Bayesian-updated Correlated Process

1. No growing trend for catastrophic losses

(1) No Bayesian-updated serial correlation

$$L_t = L_0 + \varepsilon_t, t = 1, 2, \dots, T$$

$$\varepsilon_t \sim (0, \sigma^2(\varepsilon))$$

$$\sum_{t=1}^T L_t = \sum_{t=1}^T (L_0 + \varepsilon_t) = T \cdot L_0 + T \cdot \sum_{t=1}^T \varepsilon_t$$

$$\sigma^2\left(\sum_{t=1}^T L_t\right) = T \cdot \sigma^2(\varepsilon) \quad (3.22)$$

(2) With Bayesian-updated serial correlation

$$L_t = E_{t-1}(L_t) + \varepsilon_t, \varepsilon_t \sim (0, \sigma^2(\varepsilon))$$

$$E_{t-1}(L_t) = w \cdot L_{t-1} + (1-w) \cdot E_{t-2}(L_{t-1})$$

$$t = 1, 2, \dots, T$$

$$\Rightarrow L_t = L_0 + w \cdot \sum_{i=1}^{t-1} \varepsilon_i + \varepsilon_t$$

This result can be proved by mathematical induction.

$$L_1 = L_0 + \varepsilon_1$$

Assume that $L_k = L_0 + w \cdot \sum_{i=1}^{k-1} \varepsilon_i + \varepsilon_k$ is true.

$$\begin{aligned}
L_{k+1} &= E_k(L_{k+1}) + \varepsilon_{k+1} \\
&= w \cdot L_k + (1-w) \cdot E_{k-1}(L_k) + \varepsilon_{k+1} \\
&= w \cdot \left(L_0 + w \cdot \sum_{i=1}^{k-1} \varepsilon_i + \varepsilon_k \right) + (1-w) \cdot (L_k - \varepsilon_k) + \varepsilon_{k+1} \\
&= L_0 + w \cdot \sum_{i=1}^{k-1} \varepsilon_i + w \cdot \varepsilon_k + \varepsilon_{k+1} \\
&= L_0 + w \cdot \sum_{i=1}^k \varepsilon_i + \varepsilon_{k+1}
\end{aligned}$$

Thus, by mathematical induction, $L_t = L_0 + w \cdot \sum_{i=1}^{t-1} \varepsilon_i + \varepsilon_t$ are true for $t = 1, 2, \dots, T$

Based on the value of L_t , we can derive the value of $\sum_{t=1}^T L_t$ and $\sigma_{Bayes}^2 \left(\sum_{t=1}^T L_t \right)$

$$\begin{aligned}
&\sum_{t=1}^T L_t \\
&= \sum_{t=1}^T \left(L_0 + w \cdot \sum_{i=1}^{t-1} \varepsilon_i + \varepsilon_t \right) \\
&= T \cdot L_0 + w \cdot \left[\sum_{t=1}^T \left(\sum_{i=1}^{t-1} \varepsilon_i \right) \right] + \sum_{t=1}^T \varepsilon_t
\end{aligned}$$

$$\begin{aligned}
&\sigma_{Bayes}^2 \left(\sum_{t=1}^T L_t \right) \\
&= \sigma^2 \left(\sum_{t=1}^T \varepsilon_t \right) + w^2 \cdot \sigma^2 \left(\sum_{t=1}^T \left(\sum_{i=1}^{t-1} \varepsilon_i \right) \right) + 2w \cdot \text{cov} \left(\sum_{t=1}^T \left(\sum_{i=1}^{t-1} \varepsilon_i \right), \sum_{t=1}^T \varepsilon_t \right) \\
&= T \cdot \sigma^2(\varepsilon) + w^2 \cdot \left[\sum_{i=1}^{T-1} i + 2 \cdot \sum_{i=1}^{T-2} i \cdot (T-1-i) \right] \cdot \sigma^2(\varepsilon) + 2w \cdot \left[\frac{T(T-1)}{2} \right] \cdot \sigma^2(\varepsilon) \\
&= T \cdot \sigma^2(\varepsilon) + w^2 \cdot \left[\frac{T(T-1)}{2} + (T-1)^2(T-2) - \frac{T(T-1)(T-2)}{3} \right] \cdot \sigma^2(\varepsilon) \\
&\quad + w \cdot (T(T-1)) \cdot \sigma^2(\varepsilon) \\
&= T \cdot \sigma^2(\varepsilon) + w^2 \cdot \frac{T-1}{6} \cdot \left[4 \left(T - \frac{11}{8} \right)^2 + \frac{71}{16} \right] \cdot \sigma^2(\varepsilon) + w \cdot (T(T-1)) \cdot \sigma^2(\varepsilon) \quad (3.23)
\end{aligned}$$

Comparing the values of (3.22) and (3.23) indicates that $\sigma_{Bayes}^2\left(\sum_{t=1}^T L_t\right) \geq \sigma^2\left(\sum_{t=1}^T L_t\right)$, with equality only when $T=1$.

As we apply variance in the presence of Bayesian-updated serial correlation to price insurance premium, annual premiums will increase with longer timescales. This result can be verified by obtaining the derivative of the volatility with respect to the timescale.

$$\frac{\partial \frac{\sigma_{Bayes}\left(\sum_{t=1}^T L_t\right)}{T}}{\partial T} = \frac{1 + w^2 \left[2\left(T - \frac{5}{4}\right)^2 + \frac{17}{24} \right] + w(2T-1)}{2\sqrt{T + \frac{T-1}{6} \cdot \left[4\left(T - \frac{11}{8}\right)^2 + \frac{71}{16} \right]}} \cdot \sigma(\varepsilon) > 0. \text{ Taking Bayesian-updated serial}$$

correlation into account leads the annual catastrophic insurance premiums show an increasing pattern even if we do not aggregate the capital reserves for each year.

2. With growing trend for catastrophic losses

(1) No Bayesian-updated serial correlation

$$L_t = L_0 \cdot (1+a)^t + \varepsilon_t, t = 1, 2, \dots, T$$

$$\varepsilon_t \sim (0, \sigma^2(\varepsilon))$$

$$\sum_{t=1}^T L_t = \sum_{t=1}^T (L_0(1+a)^t + \varepsilon_t) = L_0 \cdot \sum_{t=1}^T (1+a)^t + \sum_{t=1}^T \varepsilon_t$$

$$\sigma^2\left(\sum_{t=1}^T L_t\right) = T \cdot \sigma^2(\varepsilon) \quad (3.24)$$

=>The same with the case of no growing tend for catastrophic losses

(2) With Bayesian-updated serial correlation

$$L_t = E_{t-1}(L_t) + \varepsilon_t, \varepsilon_t \sim (0, \sigma^2(\varepsilon))$$

$$E_{t-1}(L_t) = E_{t-1}(L_{t-1})(1 + a)$$

$$E_t(L_t) = w \cdot L_t + (1 - w) \cdot E_{t-1}(L_t)$$

$$t = 1, 2, \dots, T$$

$$\Rightarrow L_t = L_0 \cdot (1 + a)^t + \left[\sum_{\tau=1}^{t-1} (1 + a)^\tau \cdot w \cdot \varepsilon_\tau \right] + \varepsilon_t$$

This result can also be proved by mathematical induction.

$$L_1 = L_0 \cdot (1 + a) + \varepsilon_1$$

$$\text{Assume that } L_k = L_0 \cdot (1 + a)^k + \left[\sum_{\tau=1}^{k-1} (1 + a)^\tau \cdot w \cdot \varepsilon_\tau \right] + \varepsilon_k \text{ is true.}$$

$$\begin{aligned} L_{k+1} &= E_k(L_{k+1}) + \varepsilon_{k+1} \\ &= E_k(L_k) \cdot (1 + a) + \varepsilon_{k+1} \\ &= [w \cdot L_k + (1 - w) \cdot E_{k-1}(L_k)] \cdot (1 + a) + \varepsilon_{k+1} \\ &= [w \cdot L_k + (1 - w) \cdot (L_k - \varepsilon_k)] \cdot (1 + a) + \varepsilon_{k+1} \\ &= (L_k - \varepsilon_k) \cdot (1 + a) + w \cdot \varepsilon_k \cdot (1 + a) + \varepsilon_{k+1} \\ &= \left(L_0 \cdot (1 + a)^k + \left[\sum_{\tau=1}^{k-1} (1 + a)^\tau \cdot w \cdot \varepsilon_\tau \right] \right) \cdot (1 + a) + w \cdot \varepsilon_k \cdot (1 + a) + \varepsilon_{k+1} \\ &= L_0 \cdot (1 + a)^{k+1} + \left[\sum_{\tau=1}^k (1 + a)^\tau \cdot w \cdot \varepsilon_\tau \right] + \varepsilon_{k+1} \end{aligned}$$

Thus, by mathematical induction, $L_t = L_0 \cdot (1+a)^t + \left[\sum_{\tau=1}^{t-1} (1+a)^\tau \cdot w \cdot \varepsilon_\tau \right] + \varepsilon_t$ are true

for $t = 1, 2, \dots, T$

Based on the value of L_t , we can derive the value of $\sum_{t=1}^T L_t$ and $\sigma_{Bayes}^2 \left(\sum_{t=1}^T L_t \right)$

$$\begin{aligned} & \sum_{t=1}^T L_t \\ &= \sum_{t=1}^T \left(L_0 \cdot (1+a)^t + \left[\sum_{\tau=1}^{t-1} (1+a)^\tau \cdot w \cdot \varepsilon_\tau \right] + \varepsilon_t \right) \\ &= L_0 \sum_{t=1}^T (1+a)^t + \left[\sum_{t=1}^T \left(\sum_{\tau=1}^{t-1} (1+a)^\tau \cdot \varepsilon_\tau \right) \right] \cdot w + \sum_{t=1}^T \varepsilon_t \end{aligned}$$

$$\begin{aligned} & \sigma_{Bayes}^2 \left(\sum_{t=1}^T L_t \right) \\ &= \sigma^2 \left(\sum_{t=1}^T \varepsilon_t \right) + w^2 \cdot \sigma^2 \left(\sum_{t=1}^T \left(\sum_{\tau=1}^{t-1} (1+a)^\tau \cdot \varepsilon_\tau \right) \right) + 2w \cdot \text{cov} \left(\sum_{t=1}^T \left(\sum_{\tau=1}^{t-1} (1+a)^\tau \cdot \varepsilon_\tau \right), \sum_{t=1}^T \varepsilon_t \right) \\ &= T \cdot \sigma^2(\varepsilon) + w^2 \cdot \left[\sum_{i=1}^{T-1} (1+a)^{2i} \cdot (T-i) + 2 \cdot \sum_{i=1}^{T-2} (1+a)^{2i} \cdot \frac{(T-i) \cdot (T-1-i)}{2} \right] \cdot \sigma^2(\varepsilon) \\ &+ 2w \cdot \left[\sum_{\tau=1}^{T-1} (1+a)^i \cdot (T-i) \right] \cdot \sigma^2(\varepsilon) \tag{3.25} \end{aligned}$$

A Comparison of the values between (3.24) and (3.25) indicates that

$$\sigma_{Bayes}^2 \left(\sum_{t=1}^T L_t \right) \geq \sigma^2 \left(\sum_{t=1}^T L_t \right), \text{ with equality only when } T=1.$$

3.13 Derivation of Risk Capital for Two One-period Contracts

$$\begin{aligned}
& E^Q \left[(L_1 - P_1)^+ \right] \\
&= E^Q \left[L_1 I_{(L_1 > \mu_1)} \right] - \mu_1 E^Q \left[I_{(L_1 > \mu_1)} \right], L_1 \sim N(\mu_1, \sigma) \\
&= \int_0^\infty (\mu_1 + \sigma Z^Q) \phi(Z^Q) dZ^Q - \frac{\mu_1}{2} \\
&= \frac{\sigma}{\sqrt{2\pi}}
\end{aligned} \tag{3.26}$$

$$\begin{aligned}
& E^Q \left[(L_2^N | L_1^R - P_2^N)^+ \right] \\
&= E^Q \left[L_2^N | L_1^R I_{(L_2^N | L_1^R > \mu_2^N)} \right] - \mu_2^N E^Q \left[I_{(L_2^N | L_1^R > \mu_2^N)} \right], L_2^N | L_1 \sim N(\mu_2^N, \sigma) \\
&= \int_0^\infty (\mu_2^N + \sigma Z^Q) \phi(Z^Q) dZ^Q - \frac{\mu_2^N}{2} \\
&= \frac{\sigma}{\sqrt{2\pi}}
\end{aligned} \tag{3.27}$$

Table 1.1: Fair Values of Indemnity-based, PCS-index, and Hybrid-trigger CAT bonds and Their Differences in Case (1) $a_i = 0.02, \alpha_i = 0.05, \bar{\alpha}_i = 0.07$

Threshold	0.2	0.6	1	1.6	2
Correlation	Panel A: Value of Indemnity-based CAT Bond (VI)				
0.1	21.9717	22.2245	22.3302	22.3844	22.4011
0.5	21.9792	22.2316	22.3245	22.3838	22.3989
1	21.9711	22.2292	22.3275	22.3846	22.402
	Panel B: Value of PCS-index CAT Bond (VPCS)				
0.1	22.189	22.3809	22.4126	22.4244	22.4277
0.5	22.1878	22.3764	22.4126	22.425	22.4283
1	22.1844	22.3794	22.4131	22.4246	22.4279
	Panel C: Value of Hybrid-trigger CAT Bond (VH)				
0.1	21.9102	22.1515	22.2647	22.3491	22.3763
0.5	21.9157	22.1568	22.2663	22.3454	22.371
1	21.9085	22.1479	22.2655	22.3508	22.3762
	Panel D: Difference of the Values of CAT Bonds (VI-VPCS)				
0.1	-0.2173	-0.1564	-0.0824	-0.04	-0.0266
0.5	-0.2086	-0.1448	-0.0881	-0.0412	-0.0294
1	-0.2133	-0.1502	-0.0856	-0.04	-0.0259
	Panel E: Difference of the Values of CAT Bonds (VI-VH)				
0.1	0.0615	0.073	0.0655	0.0353	0.0248
0.5	0.0635	0.0748	0.0582	0.0384	0.0279
1	0.0626	0.0813	0.062	0.0338	0.0258

Table 1.2: Fair Values of Indemnity-based, PCS-index, and Hybrid-trigger CAT bonds and Their Differences in Case (2) $a_i = 0.07, \alpha_i = 0.05, \bar{\alpha}_i = 0.08$

Threshold	0.2	0.6	1	1.6	2
Correlation	Panel A: Value of Indemnity-based CAT Bond (VI)				
0.1	21.9601	22.2224	22.3234	22.3848	22.4029
0.5	21.9535	22.225	22.3204	22.382	22.4001
1	21.957	22.2202	22.324	22.3832	22.3996
	Panel B: Value of PCS-index CAT Bond (VPCS)				
0.1	21.8933	22.1404	22.2601	22.3425	22.3727
0.5	21.8878	22.1391	22.2582	22.3379	22.3689
1	21.8906	22.1402	22.2592	22.3436	22.3711
	Panel C: Value of Hybrid-trigger CAT Bond (VH)				
0.1	21.8737	22.1053	22.2294	22.3222	22.3577
0.5	21.8666	22.1083	22.2351	22.3183	22.3531
1	21.8731	22.1077	22.2324	22.3256	22.3565
	Panel D: Difference of the Values of CAT Bonds (VI-VPCS)				
0.1	0.0668	0.082	0.0633	0.0423	0.0302
0.5	0.0657	0.0859	0.0622	0.0441	0.0312
1	0.0664	0.08	0.0648	0.0396	0.0285
	Panel E: Difference of the Values of CAT Bonds (VI-VH)				
0.1	0.0864	0.1171	0.094	0.0626	0.0452
0.5	0.0869	0.1167	0.0853	0.0637	0.047
1	0.0839	0.1125	0.0916	0.0576	0.0431

Table 1.3: Fair Values of Indemnity-based, Model-based, and Hybrid-trigger CAT bonds and Their Differences in Case (3) ($\mu_1 = 2, \mu_2 = 1.8, \alpha_i = 0.05, \bar{\alpha}_i = 0.08$)

Threshold	0.2	0.6	1	1.6	2
Correlation	Panel A: Value of Indemnity-based CAT Bond (VI)				
0.1	21.96	22.2255	22.3315	22.3847	22.4029
0.5	21.9551	22.229	22.328	22.3827	22.3989
1	21.9557	22.2222	22.3251	22.3854	22.401
	Panel B: Value of Model-based CAT Bond (VM)				
0.1	21.9099	22.1599	22.2768	22.3486	22.3752
0.5	21.9062	22.1696	22.279	22.3543	22.3782
1	21.9052	22.1589	22.273	22.3529	22.3792
	Panel C: Value of Hybrid-trigger CAT Bond (VH)				
0.1	21.8743	22.1138	22.2362	22.3273	22.3591
0.5	21.8735	22.1075	22.2403	22.3273	22.3554
1	21.8711	22.1093	22.2311	22.3215	22.3566
	Panel D: Difference of the Values of CAT Bonds (VI-VM)				
0.1	0.0501	0.0656	0.0547	0.0361	0.0277
0.5	0.0489	0.0594	0.049	0.0284	0.0207
1	0.0505	0.0633	0.0521	0.0325	0.0218
	Panel E: Difference of the Values of CAT Bonds (VI-VH)				
0.1	0.0857	0.1117	0.0953	0.0574	0.0438
0.5	0.0816	0.1215	0.0877	0.0554	0.0435
1	0.0846	0.1129	0.094	0.0639	0.0444

Table 1.4: Fair Values of Indemnity-based, Model-based, and Hybrid-trigger CAT bonds and Their Differences in Case (4) ($\mu_1 = 2, \mu_2 = 2.2, \alpha_i = 0.05, \bar{\alpha}_i = 0.02$)

Threshold	0.2	0.6	1	1.6	2
Correlation	Panel A: Value of Indemnity-based CAT Bond (VI)				
0.1	21.9505	22.2176	22.3185	22.3777	22.3992
0.5	21.9472	22.2316	22.3278	22.385	22.4029
1	21.9516	22.2212	22.3206	22.3813	22.399
	Panel B: Value of Model-based CAT Bond (VM)				
0.1	22.1247	22.349	22.3991	22.421	22.4249
0.5	22.1222	22.355	22.4042	22.422	22.4259
1	22.1321	22.3509	22.4005	22.4192	22.4241
	Panel C: Value of Hybrid-trigger CAT Bond (VH)				
0.1	22.1735	22.3721	22.4115	22.425	22.4271
0.5	22.1896	22.3781	22.4135	22.4259	22.4277
1	22.1791	22.3749	22.4112	22.4233	22.4267
	Panel D: Difference of the Values of CAT Bonds (VI-VM)				
0.1	-0.1742	-0.1314	-0.0806	-0.0433	-0.0257
0.5	-0.175	-0.1234	-0.0764	-0.037	-0.023
1	-0.1805	-0.1297	-0.0799	-0.0379	-0.0251
	Panel E: Difference of the Values of CAT Bonds (VI-VH)				
0.1	-0.223	-0.1545	-0.093	-0.0473	-0.0279
0.5	-0.2424	-0.1465	-0.0857	-0.0409	-0.0248
1	-0.2275	-0.1537	-0.0906	-0.042	-0.0277

Table 1.5: Fair Values of Indemnity-based, Model-based, and Hybrid-trigger CAT bonds and Their Differences in Case (5) ($\mu_1 = 2, \mu_2 = 1.8, \alpha_i = 0.05, \bar{\alpha}_i = 0.02$)

Threshold	0.2	0.6	1	1.6	2
Correlation	Panel A: Value of Indemnity-based CAT Bond (VI)				
0.1	21.9555	22.2282	22.3217	22.3821	22.4
0.5	21.962	22.2231	22.328	22.3829	22.4018
1	21.9633	22.2217	22.3239	22.3815	22.4008
	Panel B: Value of Model-based CAT Bond (VM)				
0.1	22.2369	22.3996	22.422	22.4288	22.4293
0.5	22.2356	22.3908	22.419	22.4265	22.4289
1	22.2276	22.3938	22.4179	22.427	22.4284
	Panel C: Value of Hybrid-trigger CAT Bond (VH)				
0.1	22.1852	22.3756	22.4113	22.4244	22.4275
0.5	22.1807	22.3769	22.4115	22.4245	22.4277
1	22.1804	22.3742	22.4119	22.4235	22.427
	Panel D: Difference of the Values of CAT Bonds (VI-VM)				
0.1	-0.2814	-0.1714	-0.1003	-0.0467	-0.0293
0.5	-0.2736	-0.1677	-0.091	-0.0436	-0.0271
1	-0.2643	-0.1721	-0.094	-0.0455	-0.0276
	Panel E: Difference of the Values of CAT Bonds (VI-VH)				
0.1	-0.2297	-0.1474	-0.0896	-0.0423	-0.0275
0.5	-0.2187	-0.1538	-0.0835	-0.0416	-0.0259
1	-0.2171	-0.1525	-0.088	-0.042	-0.0262

Table 1.6: Fair Values of Indemnity-based, Model-based, and Hybrid-trigger CAT bonds and Their Differences in Case (6) ($\mu_1 = 2, \mu_2 = 2.2, \alpha_i = 0.05, \bar{\alpha}_i = 0.08$)

Threshold	0.2	0.6	1	1.6	2
Correlation	Panel A: Value of Indemnity-based CAT Bond (VI)				
0.1	21.9503	22.2226	22.3194	22.3805	22.3971
0.5	21.9624	22.2267	22.3232	22.3848	22.3996
1	21.9468	22.2094	22.3159	22.3809	22.4002
	Panel B: Value of Model-based CAT Bond (VM)				
0.1	21.848	22.054	22.1836	22.289	22.3273
0.5	21.8503	22.0608	22.1838	22.2879	22.3258
1	21.8401	22.0498	22.1728	22.2794	22.3181
	Panel C: Value of Hybrid-trigger CAT Bond (VH)				
0.1	21.8671	22.1066	22.2324	22.3198	22.3537
0.5	21.8789	22.1153	22.237	22.3212	22.3556
1	21.8642	22.0986	22.221	22.3145	22.3517
	Panel D: Difference of the Values of CAT Bonds (VI-VM)				
0.1	0.1023	0.1686	0.1358	0.0915	0.0698
0.5	0.1121	0.1659	0.1394	0.0969	0.0738
1	0.1067	0.1596	0.1431	0.1015	0.0821
	Panel E: Difference of the Values of CAT Bonds (VI-VH)				
0.1	0.0832	0.116	0.087	0.0607	0.0434
0.5	0.0835	0.1114	0.0862	0.0636	0.044
1	0.0826	0.1108	0.0949	0.0664	0.0485

PROPERTY AND CASUALTY INSURANCE INDUSTRY
2009 TOP 25 COMPANIES BY COUNTRYWIDE PREMIUM
By Line of Business
04-Homeowners Multiple Peril

RANK BY PREM	GROUP/ COMPANY CODE	GROUP/COMPANY NAME	DIRECT PREMIUMS WRITTEN	DIRECT PREMIUMS EARNED	DIRECT LOSS TO EP RATIO ¹	DIRECT LOSS & DCC TO EP RATIO ²	MARKET SHARE %	CUMULATIVE MARKET SHARE %
1	176	STATE FARM GRP	14,737,236,449	14,297,373,262	66.53	68.50	21.62	21.62
2	8	ALLSTATE INS GRP	6,837,285,776	6,862,769,439	60.40	62.40	10.03	31.65
3	212	ZURICH INS GRP	5,066,034,148	5,036,069,846	50.38	51.92	7.43	39.08
4	111	LIBERTY MUT GRP	3,461,188,432	3,316,831,558	56.33	57.76	5.08	44.16
5	3548	TRAVELERS GRP	3,082,154,511	2,983,717,008	47.20	48.56	4.52	48.68
6	200	UNITED SERV AUTOMOBILE ASSN GRP	2,969,697,662	2,806,871,503	55.20	56.29	4.36	53.04
7	140	NATIONWIDE CORP GRP	2,909,147,923	2,879,824,709	64.35	65.72	4.27	57.30
8	38	CHUBB & SON INC GRP	1,772,267,678	1,811,450,950	41.39	39.72	2.60	59.90
9	473	AMERICAN FAMILY INS GRP	1,460,618,362	1,434,633,263	68.24	69.04	2.14	62.05
10	91	HARTFORD FIRE & CAS GRP	1,122,779,740	1,107,932,897	65.91	69.38	1.65	63.69
11	280	AUTO OWNERS GRP	878,807,476	852,604,259	91.93	93.37	1.29	64.98
12	241	METROPOLITAN GRP	860,986,068	851,833,843	52.86	53.27	1.26	66.25
13	213	ERIE INS GRP	792,687,542	765,832,058	67.23	70.77	1.16	67.41
14	10064	CITIZENS PROP INS CORP	790,756,469	882,461,699	41.85	42.37	1.16	68.57
15	12	AMERICAN INTL GRP	715,528,744	502,028,806	53.27	61.34	1.05	69.62
16	761	ALLIANZ INS GRP	647,504,956	657,173,787	37.39	38.37	0.95	70.57
17	50	COUNTRY INS & FIN SERV GRP	577,090,552	559,186,343	87.51	88.57	0.85	71.42
18	4663	UNIVERSAL INS HOLDING GRP	513,340,865	492,282,608	33.80	34.60	0.75	72.17
19	1278	CALIFORNIA STATE AUTO GRP	511,857,303	495,352,273	52.69	54.69	0.75	72.92
20	88	THE HANOVER INS GRP	493,687,770	473,609,354	63.55	64.60	0.72	73.64
21	1318	AUTO CLUB ENTERPRISES INS GRP	463,117,940	444,409,266	45.52	48.59	0.68	74.32
22	28	AMICA MUT GRP	431,265,405	422,793,164	49.25	51.18	0.63	74.96
23	361	MUNICH RE GRP	393,833,029	370,328,757	46.41	46.46	0.58	75.53
24	55	AUTOMOBILE CLUB MI GRP	367,095,292	351,770,261	65.67	66.41	0.54	76.07
25	1344	ARX HOLDING CORP GRP	321,405,260	287,149,029	31.01	33.04	0.47	76.54
		INDUSTRY TOTAL	68,166,767,715	66,459,328,229	59.27	60.94	100.00	100.00

¹ (Direct losses incurred / Direct premiums earned)

² (Direct losses incurred + Direct defense and cost containment expenses incurred) / (Direct premiums earned)

Table 2.1: Statistics of Premiums and Losses of 2009 Top 25 Companies in Property and Casualty Insurance Industry. This table shows the statistics of premiums and losses of 2009 top 25 companies in property and casualty insurance industry, collected by NAIC (National Association of Insurance Commissioners). The ratios of losses to premiums are 59.27% and 60.94%, based on different definitions of losses. The former is defined by direct losses incurred divided by direct premiums earned while the latter is defined by the sum of direct losses incurred and direct defense and cost containment expenses incurred divided by direct premium earned.

Source: http://www.naic.org/research_home.htm

Table 3.1: Climate Change Effect on the Statistics of the Simulated Losses					
p=0.01,T=20, L=1	Simulated	Exact	Simulated	Exact	
Climate change factor (a)	0	0	0.05	0.05	%change
Expected value	0.2018	0.2	0.2428	0.2422	20.32%
Standard deviation	0.4482	0.445	0.5465		21.93%
Skewness	2.2029	2.2024	2.2903		3.97%
Kurtosis	7.7498	7.7505	8.2858		6.92%
VaR (95%)	1	1	1.3829		38.29%
VaR (97.5%)	1	1	1.6533		65.33%
VaR (99%)	2	2	2.3314		16.57%
ES (95%)	1.3754	1.3581	1.8622		35.39%
ES (97.5%)	1.7508	1.7163	2.2166		26.60%
ES (99%)	2.107	2.1048	2.7279		29.47%
Prob(Loss>1.0)	0.0177	0.0169	0.1663	0.6207	839.55%
Prob(Loss>1.5)	0.0177	0.0169	0.0416	0.3554	135.03%
Prob(Loss>2.0)	0.001	0.001	0.0165	0.1859	1550.00%
Prob(Loss>2.5)	0.001	0.001	0.0072	0.0915	620.00%
Prob(Loss>3.0)	0.0001	0	0.0014	0.043	1300.00%

The exceedance probabilities of the loss with climate change can not be derived directly; thus the Chernoff bounds are derived for the thresholds indicated in the table.

Table 3.2: Impact of Climate Change Uncertainty on the Statistics of the Simulated Losses

p=0.01, T=20, L=1	(1)	(2)	(3)	(4)	(5)
Climate change uncertainty	0.05	0.025~0.075	0.0~0.1	(2)-(1)	(3)-(1)
Expected value	0.2428	0.2443	0.2483	0.62%	2.27%
Standard deviation	0.5465	0.5523	0.562	1.06%	2.84%
Skewness	2.2903	2.3246	2.331	1.50%	1.78%
Kurtosisness	8.2858	8.5483	8.5564	3.17%	3.27%
VaR (95%)	1.3829	1.4019	1.4406	1.37%	4.17%
VaR (97.5%)	1.6533	1.6986	1.69	2.74%	2.22%
VaR (99%)	2.3314	2.3478	2.3539	0.70%	0.97%
ES (95%)	1.8622	1.8963	1.9482	1.83%	4.62%
ES (97.5%)	2.2166	2.2529	2.302	1.64%	3.85%
ES (99%)	2.7279	2.7808	2.8221	1.94%	3.45%
Prob(Loss>1.00)	0.1663	0.1821	0.1824	9.50%	9.68%
Prob(Loss>2.00)	0.0165	0.0166	0.0164	0.61%	-0.61%
Prob(Loss>3.00)	0.0014	0.002	0.0022	42.86%	57.14%
Prob(Loss>4.00)	0.0001	0.0002	0.0003	100.00%	200.00%

Table 3.3: Time Scale and Cost/Benefit of Optimal Mitigation

Discount Rate=0%						
Time Scale	1	2	5	10	15	20
optimal mitigation levels	0	0.05	0.15	0.25	0.5	0.5
TC with optimal mitigation	0.0102	0.0228	0.0557	0.105	0.1467	0.1846
benefit from optimal mitigation	0	0.0095	0.0275	0.0701	0.147	0.21
optimal mitigation cost	0	0.0028	0.0099	0.0195	0.0563	0.0563
net benefit from optimal mitigation	0	0.0067	0.0176	0.0505	0.0908	0.1537
TC with no mitigation	0.0102	0.0296	0.0733	0.1555	0.2375	0.3383
RNB from optimal mitigation	0.00%	22.71%	23.96%	32.49%	38.21%	45.44%
Discount Rate=5%						
Time Scale	1	5	6	10	15	20
optimal mitigation levels	0	0	0.05	0.15	0.35	0.35
TC with optimal mitigation	0.0097	0.0462	0.0535	0.0828	0.1097	0.1268
benefit from optimal mitigation	0	0	0.0041	0.014	0.0466	0.0621
optimal mitigation cost	0	0	0.0028	0.0099	0.0319	0.0319
net benefit from optimal mitigation	0	0	0.0014	0.0041	0.0147	0.0302
TC with no mitigation	0.0097	0.0462	0.0548	0.0869	0.1244	0.1569
RNB from optimal mitigation	0.00%	0.00%	2.49%	4.73%	11.80%	19.23%

Table 3.4: Total Cost with No Mitigation No Climate Change, with Optimal Mitigation No Climate Change, with No Mitigation with Climate Change, with Optimal Mitigation and Climate Change for Different Discount Rates

Time Scale	1	5	10	15	20
Discount Rate=0%					
TC with no mitigation no CC	0.0102	0.0729	0.1523	0.2237	0.2991
TC with optimal mitigation no CC	0.0102	0.0509	0.0966	0.1316	0.1573
TC with no mitigation with CC	0.0102	0.0733	0.1555	0.2375	0.3383
TC with optimal mitigation with CC	0.0102	0.0557	0.1050	0.1467	0.1846
Discount Rate=5%					
TC with no mitigation no CC	0.0094	0.0443	0.0793	0.1094	0.1329
TC with optimal mitigation no CC	0.0094	0.0443	0.0770	0.0996	0.1141
TC with no mitigation with CC	0.0097	0.0462	0.0869	0.1244	0.1569
TC with optimal mitigation with CC	0.0097	0.0462	0.0828	0.1097	0.1268

Table 3.5: Climate Change Effect, Mitigation Effect, and Aggregate Effect for Different Discount Rates

Time Scale	1	5	10	15	20
Discount Rate=0%					
Climate Change Effect	0.00%	0.55%	2.10%	6.17%	13.11%
Mitigation Effect	0.00%	-24.01%	-32.48%	-38.23%	-45.43%
Aggregate Effect	0.00%	-23.59%	-31.06%	-34.42%	-38.28%
Discount Rate=5%					
Climate Change Effect	3.19%	4.29%	9.58%	13.71%	18.06%
Mitigation Effect	0.00%	0.00%	-4.72%	-11.82%	-19.18%
Aggregate Effect	3.19%	4.29%	4.41%	0.27%	-4.59%

Table 3.6: Statistics of Simulated Losses for Different Climate Change Factors (a) with No Mitigation , T=20				Table 3.7: Statistics of Simulated Losses for Different Climate Change Factors (a) with Roof Mitigation (A) , T=20			
a	0	0.05	0.1	a	0	0.05	0.1
Expected Value	0.83	1.01	1.28	Expected Value	0.61	0.74	0.95
Std	0.51	0.63	0.84	Std	0.41	0.50	0.67
Skewness	0.86	0.92	1.05	Skewness	1.05	1.12	1.25
Kurtosis	3.81	3.98	4.31	Kurtosis	4.28	4.54	4.95
VaR (95%)	1.78	2.18	2.89	VaR (95%)	1.39	1.71	2.26
VaR (97.5%)	2.01	2.48	3.31	VaR (97.5%)	1.59	1.97	2.63
VaR (99%)	2.29	2.84	3.81	VaR (99%)	1.84	2.27	3.09
ES (95%)	2.10	2.59	3.47	ES (95%)	1.67	2.06	2.76
ES (97.5%)	2.31	2.86	3.85	ES (97.5%)	1.86	2.30	3.10
ES (99%)	2.59	3.21	4.33	ES (99%)	2.09	2.59	3.51
Prob(Loss>0.5)	0.69	0.77	0.82	Prob(Loss>0.5)	0.52	0.61	0.72
Prob(Loss>1)	0.32	0.44	0.56	Prob(Loss>1)	0.16	0.26	0.39
Prob(Loss>1.5)	0.10	0.20	0.35	Prob(Loss>1.5)	0.03	0.08	0.18
Prob(Loss>2)	0.02	0.07	0.18	Prob(Loss>2)	0.01	0.02	0.08
Prob(Loss>2.5)	0.01	0.02	0.09	Prob(Loss>2.5)	0.00	0.01	0.03
Prob(Loss>3)	0.00	0.01	0.04	Prob(Loss>3)	0.00	0.00	0.01

Table 3.8: Benefit/Cost of Different Mitigation Measures and Different Time Horizons for Wood Frame Building in Canaries in the Absence of Climate Change (“a”=0.00)							
Time scale	1	5	6	7	8	9	10
Benefit from Mitigation (Reduction in Expected Loss)							
Roof Mitigation (A)	0.0103	0.0544	0.0649	0.0768	0.0857	0.0986	0.1071
Opening Mitigation (B)	0.0119	0.0638	0.0786	0.09	0.1005	0.1144	0.1284
Roof & Opening (AB)	0.0198	0.1001	0.1211	0.1409	0.1607	0.1815	0.2014
Mitigation Cost							
Roof Mitigation (A)	0.092	0.092	0.092	0.092	0.092	0.092	0.092
Opening Mitigation (B)	0.0672	0.0672	0.0672	0.0672	0.0672	0.0672	0.0672
Roof & Opening (AB)	0.1592	0.1592	0.1592	0.1592	0.1592	0.1592	0.1592
Benefit-Cost Ratio							
Roof Mitigation (A)	0.1120	0.5913	0.7054	0.8348	0.9315	1.0717	1.1641
Opening Mitigation (B)	0.1771	0.9494	1.1696	1.3393	1.4955	1.7024	1.9107
Roof & Opening (AB)	0.1244	0.6288	0.7607	0.8851	1.0094	1.1401	1.2651

Table 3.9: Benefit/Cost of Different Mitigation Measures and Different Time Horizons for Wood Frame Building in Canaries in the Absence of Climate Change (“a”=0.05)							
Time scale	1	5	6	7	8	9	10
Benefit from Mitigation (Reduction in Expected Loss)							
Roof Mitigation (A)	0.0113	0.0585	0.0664	0.0802	0.0934	0.1037	0.1171
Opening Mitigation (B)	0.0126	0.0677	0.0792	0.0949	0.1093	0.1246	0.1376
Roof & Opening (AB)	0.0199	0.1066	0.1255	0.1492	0.172	0.1963	0.2161
Mitigation Cost							
Roof Mitigation (A)	0.092	0.092	0.092	0.092	0.092	0.092	0.092
Opening Mitigation (B)	0.0672	0.0672	0.0672	0.0672	0.0672	0.0672	0.0672
Roof & Opening (AB)	0.1592	0.1592	0.1592	0.1592	0.1592	0.1592	0.1592
Benefit-Cost Ratio							
Roof Mitigation (A)	0.1228	0.6359	0.7217	0.8717	1.0152	1.1272	1.2728
Opening Mitigation (B)	0.1875	1.0074	1.1786	1.4122	1.6265	1.8542	2.0476
Roof & Opening (AB)	0.1250	0.6696	0.7883	0.9372	1.0804	1.2330	1.3574

Table 3.10: Statistics for Wood Frame Building in Canaries in St Lucia for Different Settings of “a”

a	0.05	0~0.1	0.06	a	0.05	0~0.1	0.06
Expected value	1.0055	1.0291	1.0506	Prob(Loss>1.5)	0.2008	0.2141	0.226
Std	0.6254	0.6488	0.6534	Prob(Loss>1.75)	0.1214	0.1398	0.1426
Skewness	0.8905	0.9386	0.8964	Prob(Loss>2)	0.0749	0.0877	0.0861
Kurtosis	3.8287	4.0259	3.8889	Prob(Loss>2.25)	0.0416	0.0534	0.0541
VaR (95%)	2.1823	2.2521	2.2768	Prob(Loss>2.5)	0.0243	0.0316	0.0303
VaR (97.5%)	2.5791	2.559	2.6949	Prob(Loss>2.75)	0.0121	0.0177	0.0164
VaR (99%)	2.4815	2.938	2.5813	Prob(Loss>3)	0.0063	0.0073	0.0088
ES (95%)	2.8437	2.6757	2.9788	Prob(Loss>3.25)	0.0029	0.004	0.0051
ES (97.5%)	2.8225	2.9614	2.9626	Prob(Loss>3.5)	0.0013	0.0021	0.0025
ES (99%)	3.165	3.3229	3.3348	Prob(Loss>3.75)	0.0007	0.0011	0.001
Prob(Loss>0.25)	0.9128	0.8839	0.9226	Prob(Loss>4)	0.0002	0.0006	0.0005
Prob(Loss>0.5)	0.7586	0.7451	0.7795	Prob(Loss>4.25)	0.0001	0.0003	0.0003
Prob(Loss>0.75)	0.6092	0.5904	0.6187	Prob(Loss>4.5)	0	0.0002	0.0001
Prob(Loss>1)	0.4382	0.4413	0.4589	Prob(Loss>4.75)	0	0.0001	0.0001
Prob(Loss>1.25)	0.3106	0.3145	0.338	Prob(Loss>5)	0	0	0

Table 3.11: Statistics of Variability in Storm Activities over Successive 5-year Period

Statistic	All Named Storms	Cat 3 – 5 hurricanes
mean	6%	12%
95th percentile	38%	85%
65th percentile	16%	22%
50th percentile	5%	3%
35th percentile	-2%	-12%
5th percentile	-24%	-42%

Table 3.12: Percentiles of Frequency of Storm Activities, Expected Loss, Volatility of Loss, and Estimates of Climate Change Factor

	Present-Day	Values at (s-tau)=5				
		5%	35%	50%	65%	95%
Frequency of all named storms relative to present-day	1	0.76	0.98	1.05	1.16	1.38
Frequency of intense storms relative to present-day	1	0.58	0.88	1.03	1.22	1.85
expected loss over next 5 years	3376.64	2165.14	3086.28	3500.9	4050.61	5707.1
standard deviation of loss	9297.99	7325.63	8794.68	9412.44	10116.8	12026.9
Growth rate of losses (over 5 years)	0	-8.50%	-1.78%	0.73%	3.71%	11.07%
Growth rate of standard deviation (over 5 years)	0	-4.66%	-1.11%	0.24%	1.70%	5.28%

Table 3.13: Probabilities of the Occurrence of Climate Change

$P(\tau=1)$	0.1848
$P(\tau=2)$	0.1479
$P(\tau=3)$	0.2129
$P(\tau=4)$	0.2309
$P(\tau=5)$	0.2235

Table 3.14: The Distribution of “a” transformed from Five Climate Scenarios

a	-0.085	-0.0178	0.0073	0.0371	0.1107
Prob(a)	0.2	0.225	0.15	0.225	0.2

Table 3.15: Statistics, Tail Probabilities, and Annual Insurance Premiums for 5-Year Losses in Different Models

Models	Potential Growth Model			Lognormal Loss Model		
	a_step	a_linear	a_expo	present-day loss	present-day loss	5 potential climate scenarios
				Fit_EPs	Fit_E(L)& σ (L)	
Time horizon	5	5	5	5	5	5
Expected losses	16,570	16,620	16,630	16,080	16,990	18,150
Standard deviation	21,360	21,360	21,370	16,760	21,010	21,200
Skewness	2.1675	2.1456	2.1454	4.0065	6.215	5.0327
Kurtosis	8.4176	8.2913	8.2396	44.9702	124.6626	67.9487
VaR (95%)	64,210	64,180	64,190	45,510	51,750	53,830
VaR (97.5%)	77,850	77,850	77,980	58,360	68,780	70,450
VaR (99%)	93,780	93,120	93,750	78,550	96,580	98,600
ES(95%)	83,030	82,800	82,950	67,580	82,000	83,850
ES(97.5%)	95,640	95,220	95,490	83,940	104,860	106,760
ES(99%)	111,730	111,190	111,520	110,060	142,710	144,090
Prob(Loss>0.25)	0.2094	0.2115	0.2115	0.1877	0.1941	0.2271
Prob(Loss>0.5)	0.0916	0.0925	0.0923	0.0389	0.0529	0.0593
Prob(Loss>0.75)	0.0287	0.0287	0.0287	0.0113	0.0191	0.021
Prob(Loss>1)	0.007	0.0068	0.0071	0.0046	0.0089	0.0097
Prob(Loss>1.25)	0.0018	0.0018	0.0018	0.0019	0.0048	0.0049
Prob(Loss>1.5)	0.0004	0.0004	0.0004	0.001	0.0026	0.0028
Prob(Loss>1.75)	0.0001	0.0001	0.0001	0.0006	0.0016	0.0018
Prob(Loss>2)	0	0	0	0.0003	0.0011	0.0012
Annual Premium	\$5,482.89	\$5,498.33	\$5,500.39	\$4,734.06	\$5,347.47	\$5,641.83

Table 3.16: Statistics and Percentiles for 1-Year Lognormal Losses under Different Assumptions

	Empirical Data	present-day loss	present-day loss	potential climate scenarios
		Fit_EPs	Fit_E(L)& σ (L)	Fit_E(L)& σ (L)
Time horizon	1	1	1	1
Expected losses	3,377	3,200	3,310	3,640
Standard deviation	9,298	7,300	8,930	9,520
Skewness		7.66	11.83	12.05
Kurtosis		133.93	334.85	386.25
Percentile (50%)	0	0	0	120
Percentile (80%)	2,090	4,710	4,410	4,970
Percentile (90%)	6,910	8,970	8,890	9,800
Percentile (95%)	20,900	14,390	15,130	16,390
Percentile (98%)	41,230	23,840	26,160	28,440
Percentile (99%)	53,150	32,990	37,820	40,360
Percentile (99.5%)	62,780	43,670	52,510	55,040
Percentile (99.6%)	65,140	47,510	58,360	60,960
Percentile (99.8%)	72,290	60,540	77,130	80,700
Percentile (99.9%)	77,110	74,150	95,600	105,980
Percentile (99.98%)	80,480	138,130	173,940	183,120
Percentile (99.99%)	87,520	175,230	232,220	226,980

Table 3.17: Statistics, Tail Probabilities, and Annual Insurance Premiums for 5-Year Losses for Different Scenarios in Potential Growth Model

	UCS	BCS	MLS	WCS
Time horizon	5	5	5	5
Expected losses	16,630	15,590	16,460	17,940
Standard deviation	21,370	20,040	21,210	22,920
Skewness	2.1454	2.1744	2.1714	2.1227
Kurtosis	8.2396	8.5168	8.4269	8.2139
VaR (95%)	64,190	59,770	63,610	68,950
VaR (97.5%)	77,980	73,670	77,580	83,230
VaR (99%)	93,750	88,930	93,110	98,710
ES(95%)	82,950	78,160	82,510	88,450
ES(97.5%)	95,490	90,290	95,090	101,500
ES(99%)	111,520	105,150	111,170	118,830
Prob(Loss>0.25)	0.2115	0.1978	0.2079	0.2271
Prob(Loss>0.5)	0.0923	0.0783	0.0901	0.107
Prob(Loss>0.75)	0.0287	0.0229	0.0281	0.0372
Prob(Loss>1)	0.0071	0.0049	0.0066	0.0094
Prob(Loss>1.25)	0.0018	0.0012	0.0019	0.0028
Prob(Loss>1.5)	0.0004	0.0003	0.0004	0.0008
Prob(Loss>1.75)	0.0001	0.0001	0.0001	0.0002
Prob(Loss>2)	0	0	0	0.0001
Annual Premium	\$5,500.39	\$5,090.87	\$5,438.29	\$6,001.71

Table 3.18: Statistics, Tail Probabilities, and Annual Insurance Premiums for 5-Year Losses for Different Scenarios in Lognormal Loss Model

	UCS	BCS	MLS	WCS
Time horizon	5	5	5	5
Expected losses	18,150	10,770	17,420	28,420
Standard deviation	21,200	16,010	20,620	26,760
Skewness	5.0327	7.2403	4.6747	3.0625
Kurtosis	67.9487	119.0596	52.2188	26.1296
VaR (95%)	53,830	35,090	52,310	76,900
VaR (97.5%)	70,450	48,750	68,910	96,190
VaR (99%)	98,600	71,900	96,370	125,940
ES(95%)	83,850	60,830	81,840	108,990
ES(97.5%)	106,760	80,620	104,260	132,750
ES(99%)	144,090	114,450	140,640	169,180
Prob(Loss>0.25)	0.2271	0.0946	0.2045	0.4268
Prob(Loss>0.5)	0.0593	0.0233	0.0571	0.147
Prob(Loss>0.75)	0.021	0.0092	0.0192	0.0542
Prob(Loss>1)	0.0097	\$0.00	0.0089	0.0224
Prob(Loss>1.25)	0.0049	0.0026	0.0048	0.0101
Prob(Loss>1.5)	0.0028	0.0014	0.0025	0.0053
Prob(Loss>1.75)	0.0018	0.0008	0.0015	0.003
Prob(Loss>2)	0.0012	0.0006	0.0011	0.0017
Annual Premium	\$5,641.83	\$3,514.83	\$5,426.73	\$8,975.05

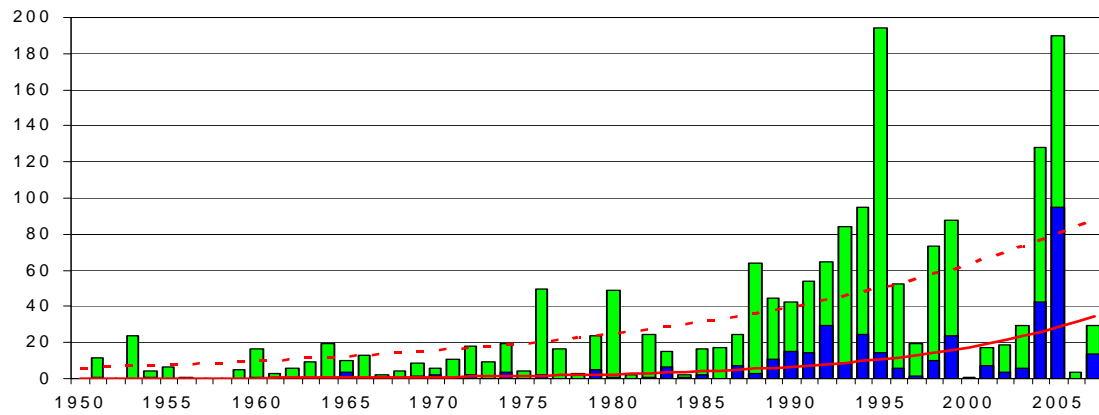
Table 3.19 Annual Premiums of Catastrophe Insurance with and without Cost of Capital and Equivalent Cost of Capital in the Absence of Climate Change ($a=0$)

Terms of contracts	1	2	3	4	5	6	7	8	9	10
Annual premium										
(no CoC)	\$48,300	\$46,600	\$44,900	\$44,700	\$44,500	\$44,300	\$44,200	\$43,600	\$43,800	\$43,600
Annual premium										
(with CoC)	\$48,300	\$50,100	\$50,400	\$51,800	\$52,900	\$53,900	\$54,800	\$55,000	\$56,100	\$56,700
Equivalent										
cost of capital	2.00%	1.56%	1.54%	1.32%	1.18%	1.09%	1.01%	1.02%	0.92%	0.93%

Table 3.20 Annual Premiums of Catastrophe Insurance with and without Cost of Capital and Equivalent Cost of Capital in the Presence of Climate Change ($a=0.019$)

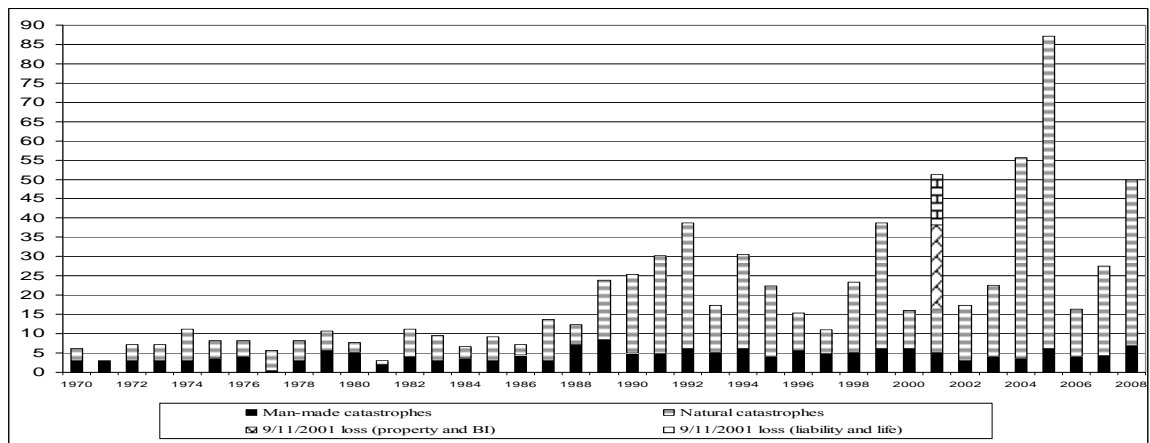
Terms of contracts	1	2	3	4	5	6	7	8	9	10
Annual premium										
(no CoC)	\$48,100	\$46,500	\$45,400	\$45,600	\$45,200	\$44,900	\$45,000	\$44,900	\$44,900	\$45,100
Annual premium										
(with CoC)	\$48,100	\$50,000	\$50,900	\$52,700	\$53,700	\$54,500	\$55,700	\$56,500	\$57,500	\$58,500
Equivalent										
cost of capital	2.00%	1.56%	1.42%	1.14%	1.04%	0.98%	0.86%	0.81%	0.74%	0.67%

Figure 1: Economic and Insured Losses from Natural Catastrophes Worldwide, 1950-2007
(in U.S.\$ billion indexed to 2007)



Source: Kunreuther and Michel-Kerjan, *At War with the Weather* (2009) - data from Munich Re

Figure 2: Worldwide Insured Losses from Catastrophes, 1970-2008
(Property and business interruption (BI); in U.S.\$ billion indexed to 2007, except 2008 which is current)



Source: Kunreuther and Michel-Kerjan, *At War with the Weather* (2009) - data from Swiss Re and Insurance Information Institute

Figure 1.1: Areas where Relative Basis Risk for the Hybrid Trigger Compared with PCS-index Trigger

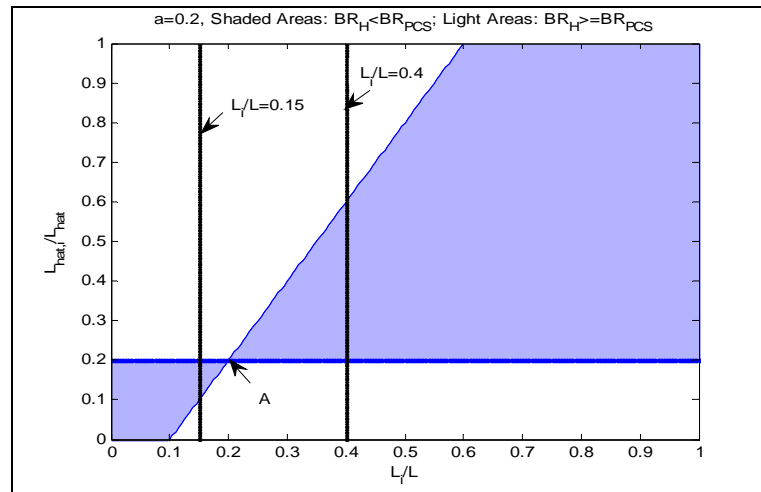


Figure 1.2: Areas where Relative Basis Risk for the Hybrid Trigger Compared with Model-based Trigger

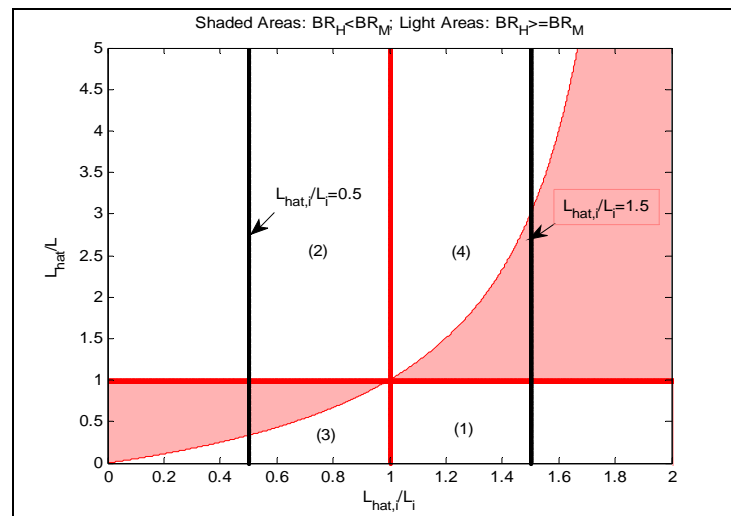


Figure 1.3: Areas where Relative Basis Risk for the Hybrid Trigger Compared with Model-based Trigger

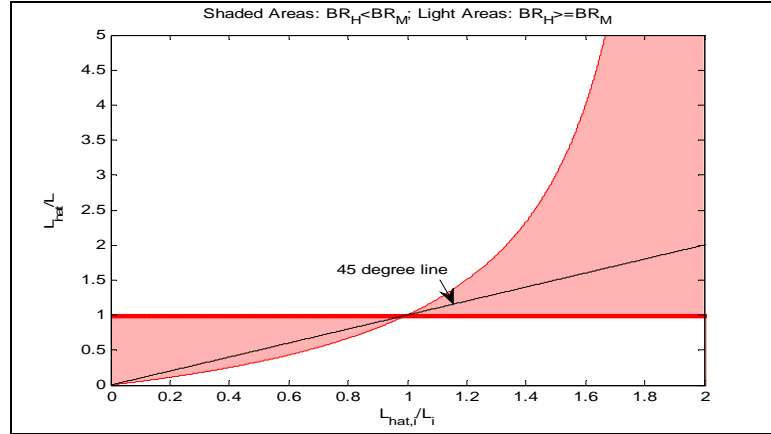


Figure 1.4: V_I-V_{PCS} for Case (1)

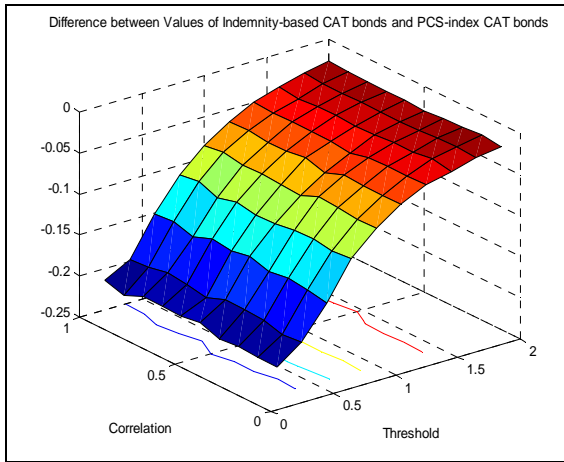


Figure 1.6: V_I-V_{PCS} for Case (2)

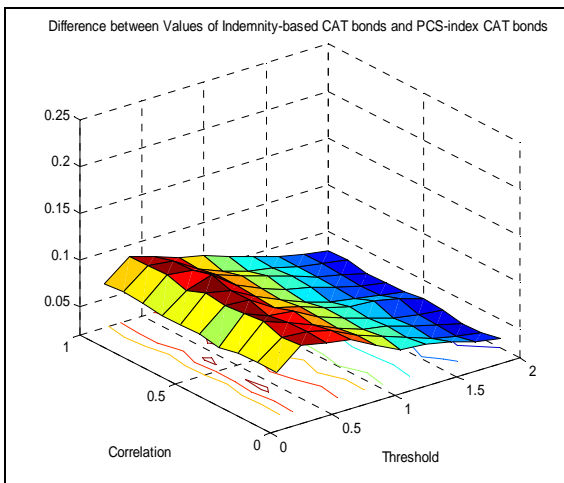


Figure 1.5: V_I-V_H for Case (1)

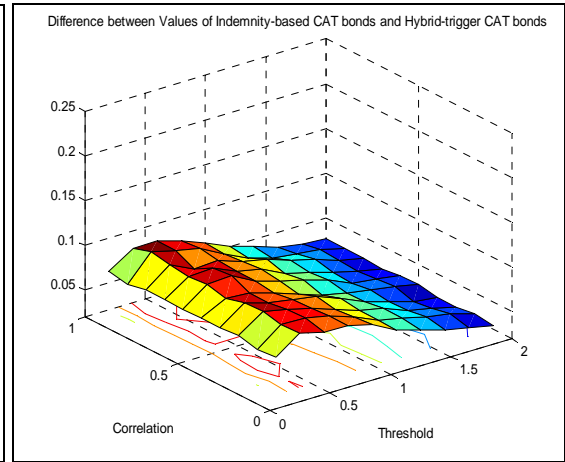


Figure 1.7: V_I-V_H for Case (2)

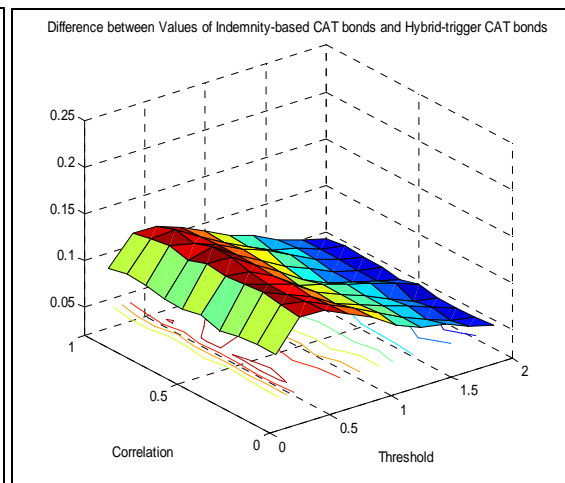


Figure 1.8: V_I in Case (3)

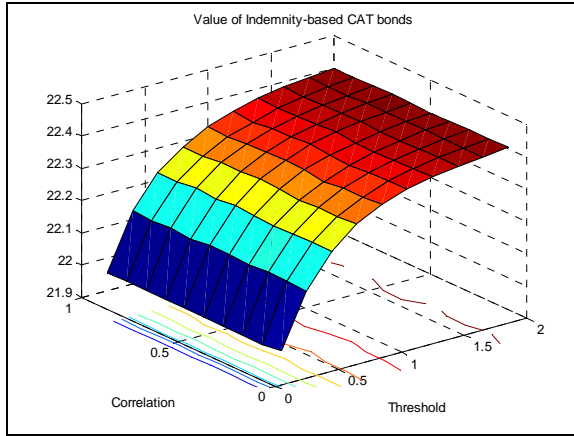


Figure 1.9: V_M in Case (3)

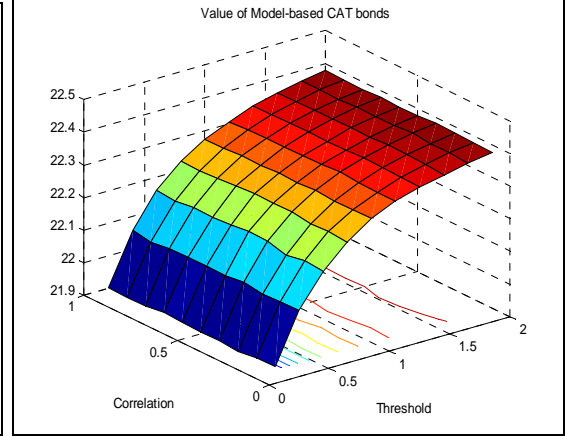


Figure 1.10: V_H in Case (3)

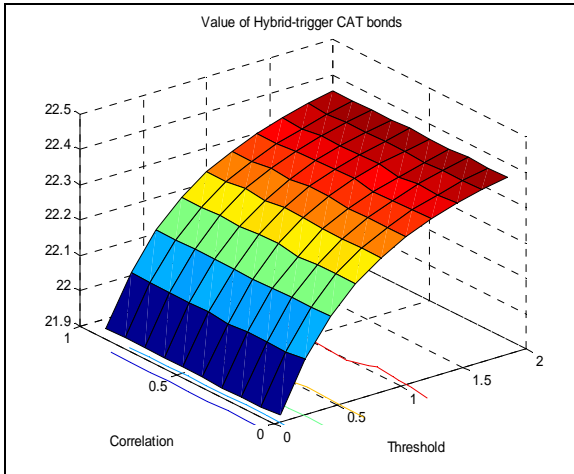


Figure 1.11: V_{PCS} in Case (3)

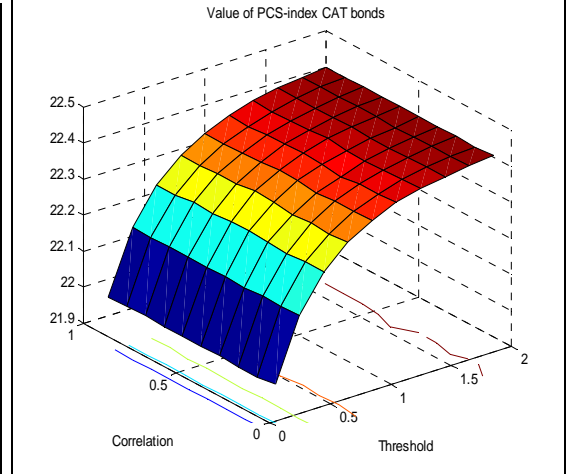


Figure 1.12: $V_I - V_M$ in Case (3)

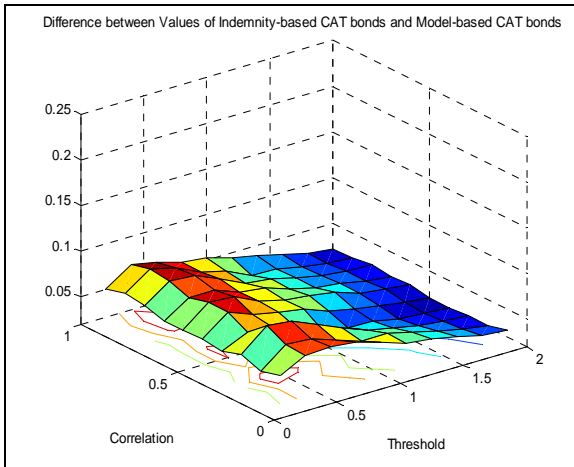


Figure 1.13: $V_I - V_H$ in Case (3)

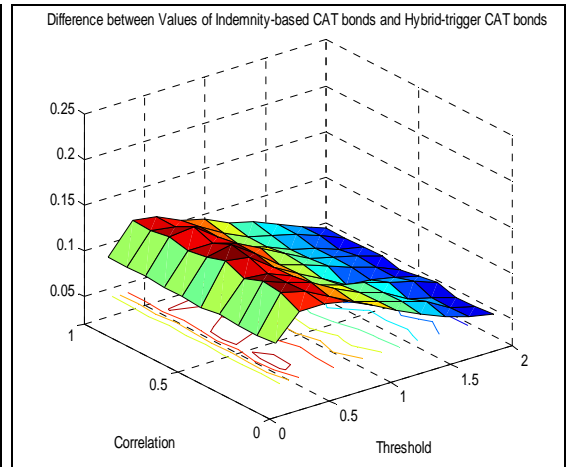


Figure 1.14: $V_I - V_M$ in Case (4)

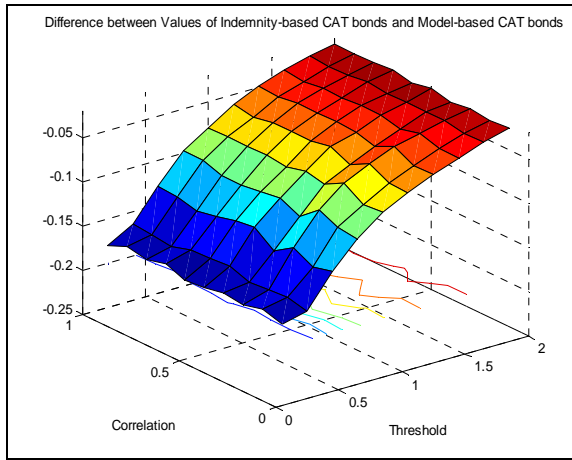


Figure 1.15: $V_I - V_H$ in Case (4)

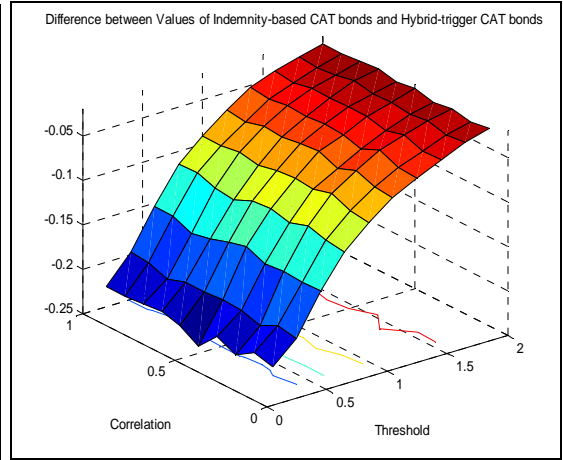


Figure 1.16: $V_I - V_M$ in Case (5)

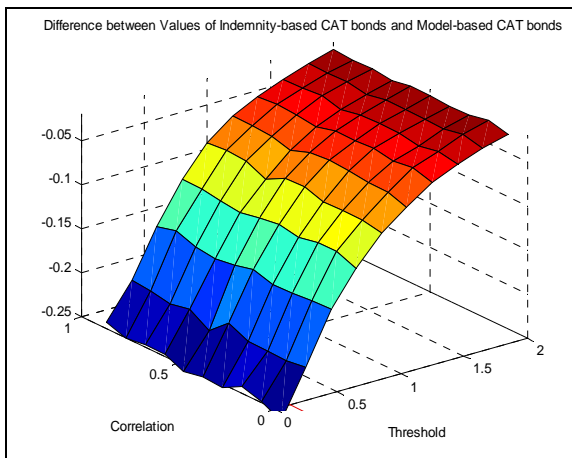


Figure 1.17: $V_I - V_H$ in Case (5)

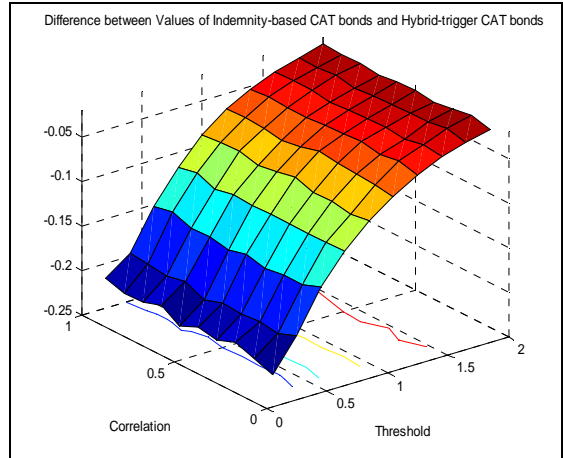


Figure 1.18: $V_I - V_M$ in Case (6)

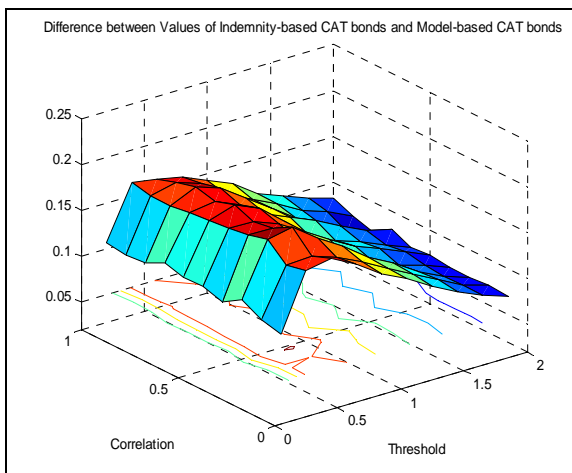


Figure 1.19: $V_I - V_H$ in Case (6)

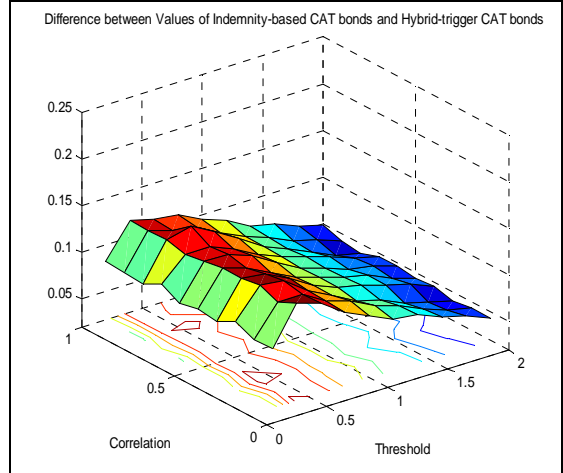


Figure 1.20: Areas where Relative Basis Risk for the Hybrid Trigger Compared with PCS-index Trigger

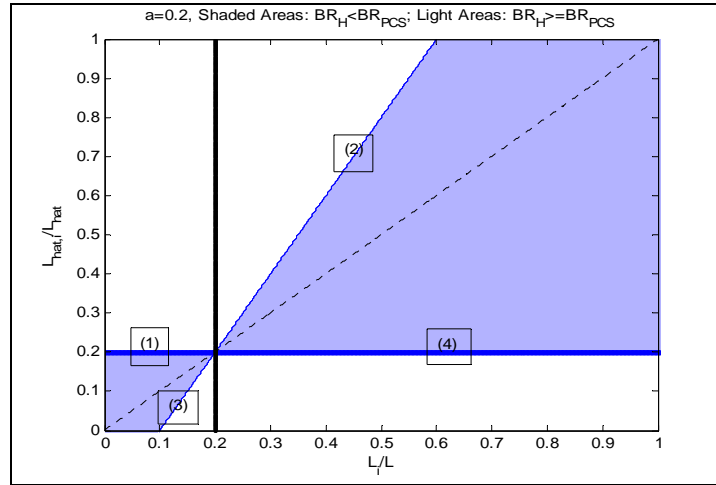


Figure 1.21: Areas where Relative Basis Risk for the Hybrid Trigger Compared with Model-based Trigger

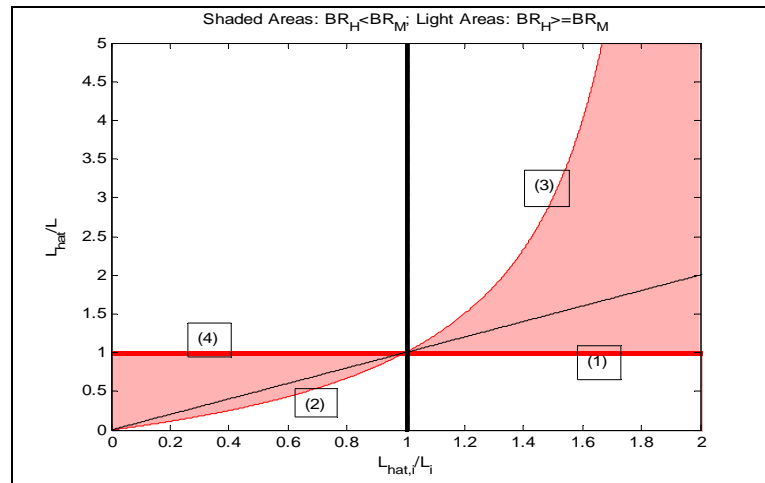


Figure 2.1: Cash Flows of Traditional Insurance Policyholders

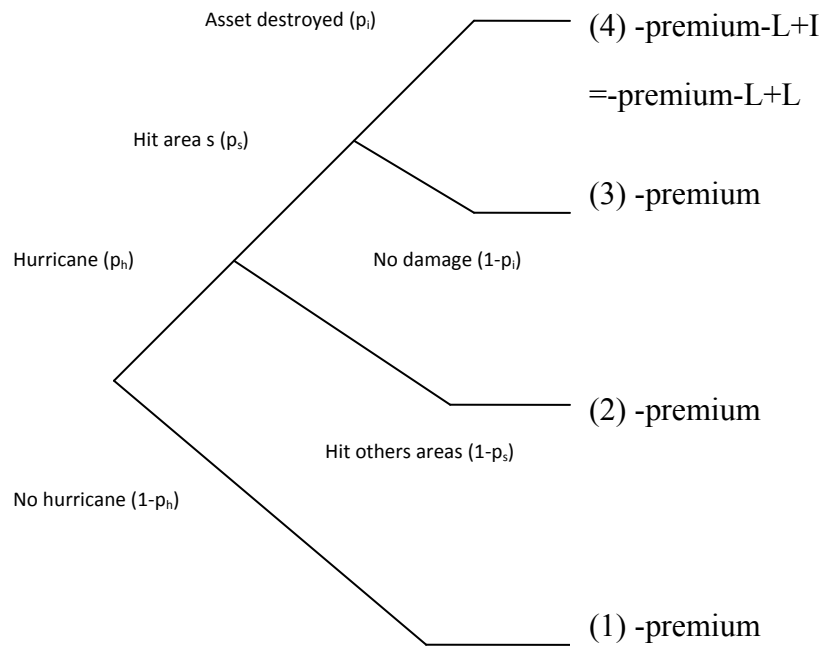
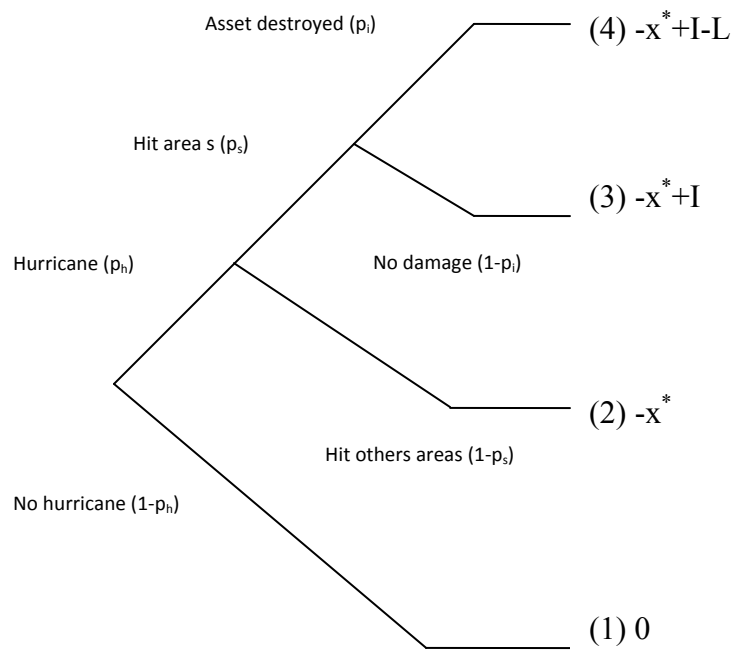


Figure 2.2: Cash Flows of Parimutuel Participants



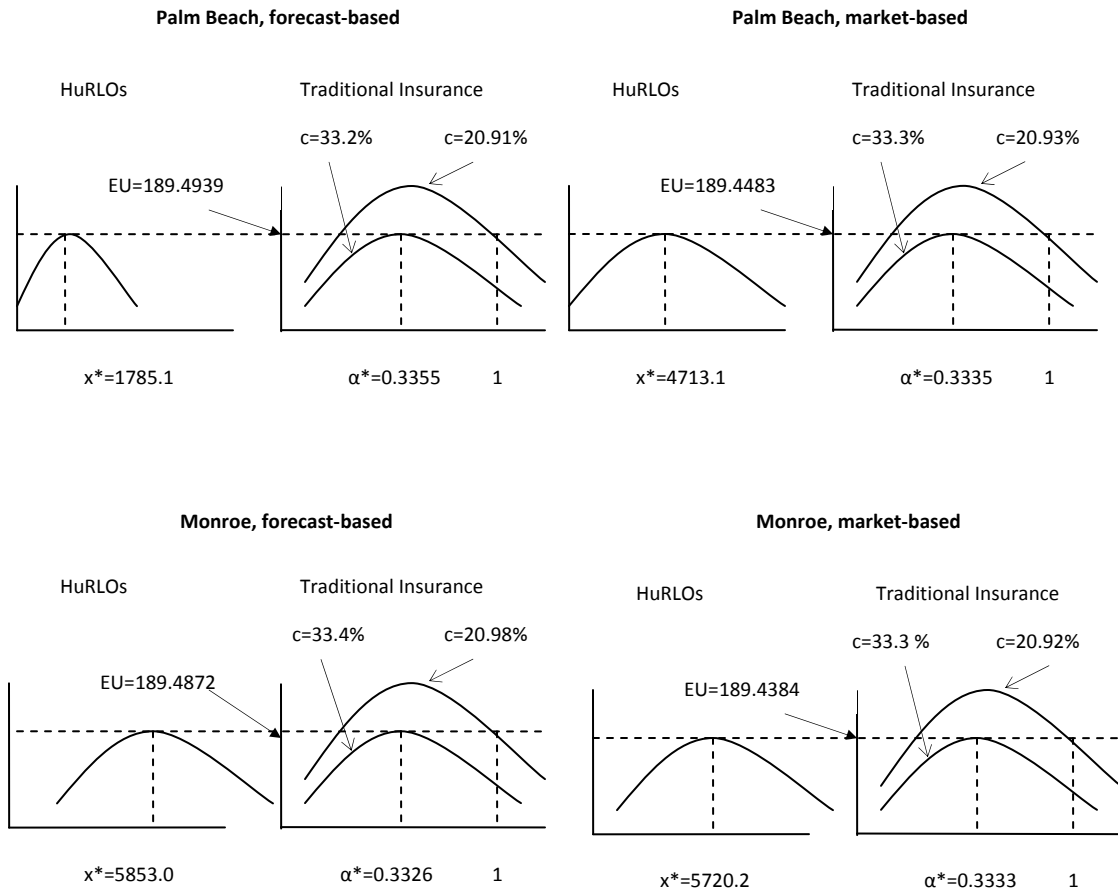


Figure 2.3: The Comparisons of the Expected Utilities for a Hedger between HuRLOs and Traditional Insurance in Four Cases. These diagrams show the comparisons of the expected utilities for a hedger between HuRLOs and traditional insurance in four cases: Palm Beach using forecast-based probability, Palm Beach using market-based probability, Monroe using forecast-based probability, and Monroe using market-based probability. For the same case, the left hand side exhibits the HuRLOs, and the right hand side demonstrates the corresponding traditional insurance. The vertical axes are the expected utility. The horizontal axis is the amount stake the individual would place (x) in HuRLOs while it is the proportion of insurance relative to full insurance (α) in traditional insurance. On the right hand side, $\alpha=1$ represents the individual being fully-insured and c denotes the equivalent transaction cost in traditional insurance contract relative to the corresponding HuRLOs. The expected utility of the individual in traditional insurance will move downward with higher transaction costs. The upper curve constrains the individual to be fully-insured while the lower curve allows the individual to be partially-insured. Transaction costs in HuRLOs are assumed to be **0%** for each stake the individual placed.

Figure 3.1 EPs for Different Climate Change Factors

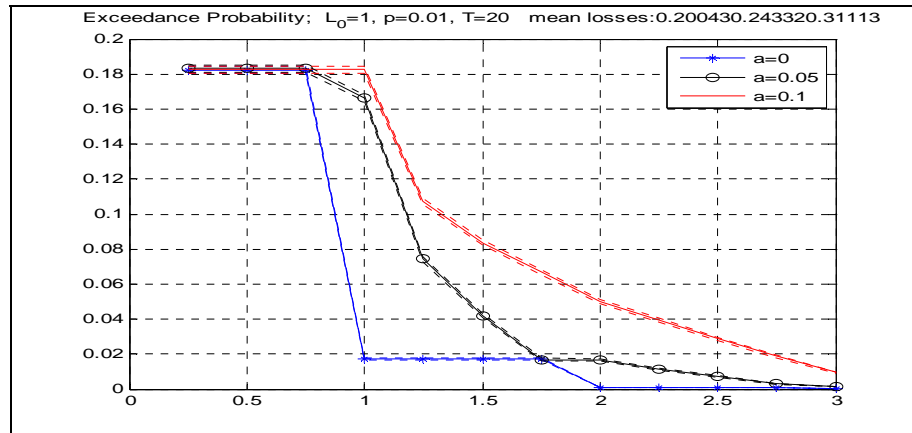


Figure 3.2

Figure 3.3

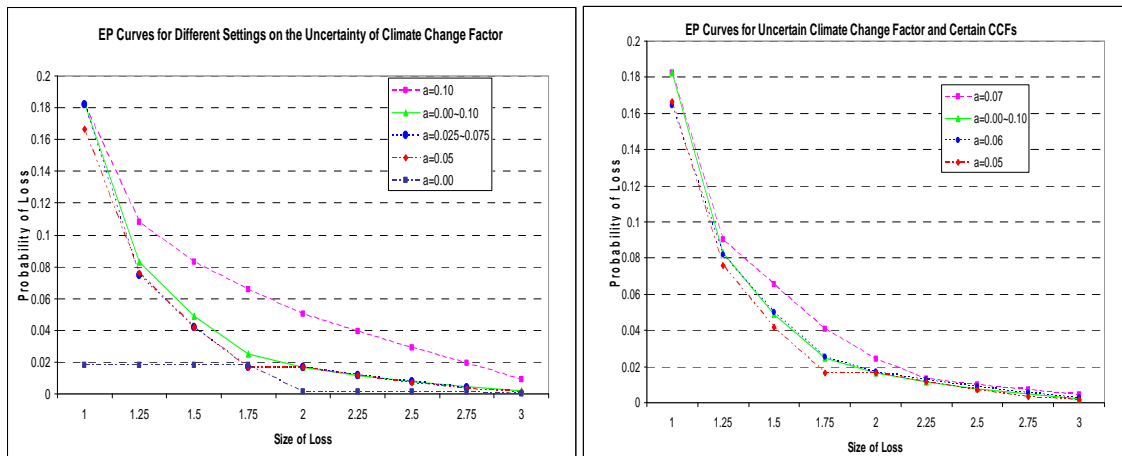


Figure 3.4: Total Costs Caused by Catastrophe v.s. Mitigation Levels

Total Cost of Catastrophe versus Mitigation Level; $L_0=1$, $p=0.01$, $a=0.05$, $T=20$, discount rate=0.1, Net Benefit=0.0089192

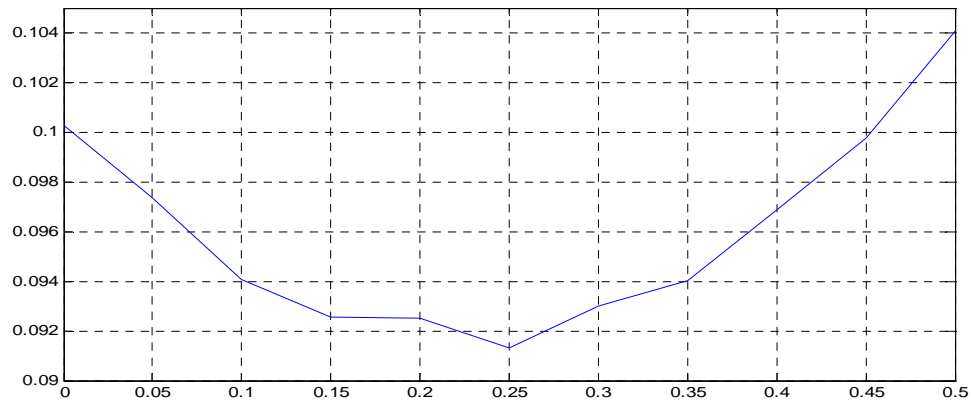


Figure 3.5

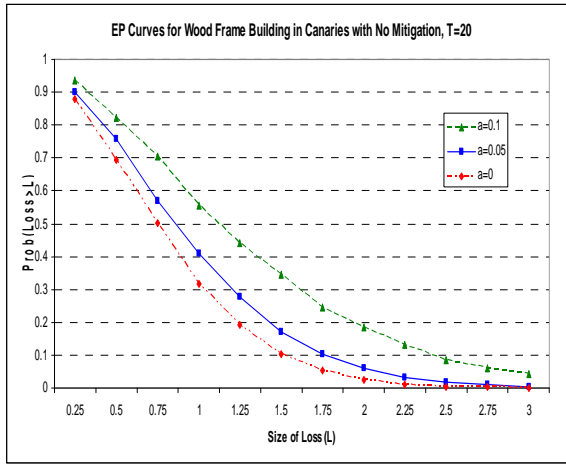


Figure 3.6

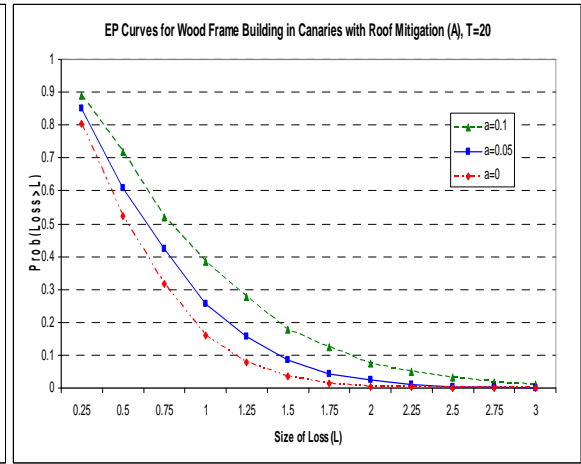


Figure 3.7

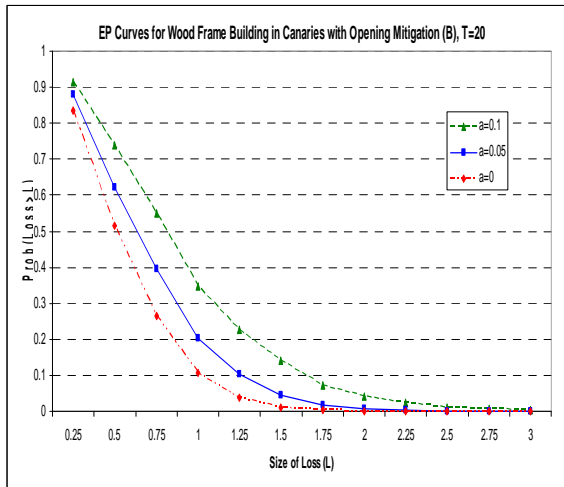


Figure 3.8

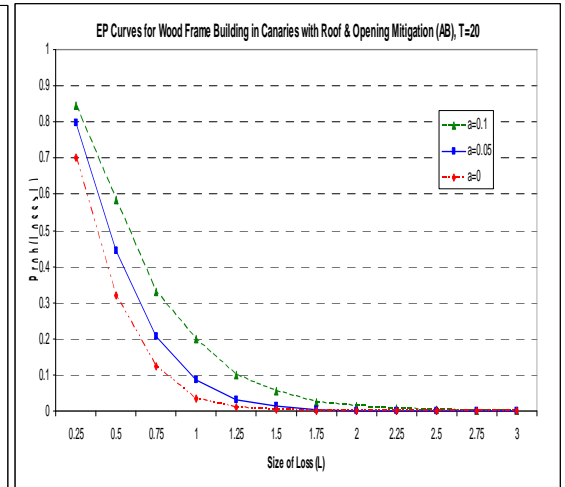


Figure 3.9

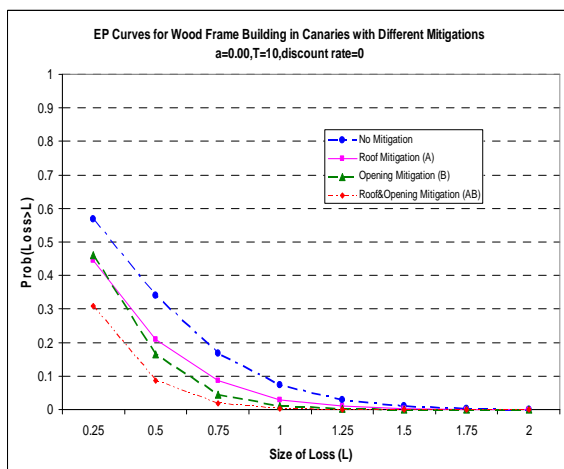


Figure 3.10

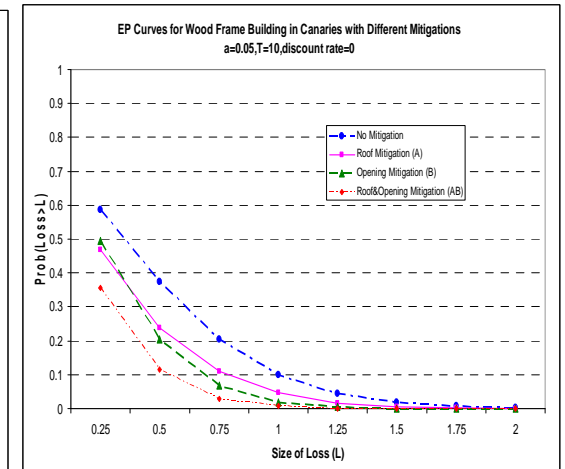


Figure 3.11

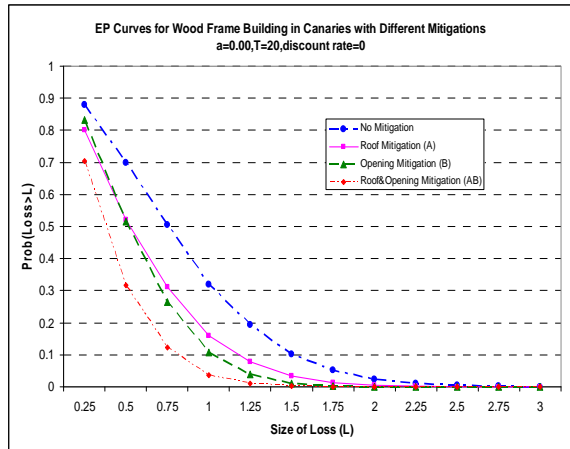


Figure 3.12

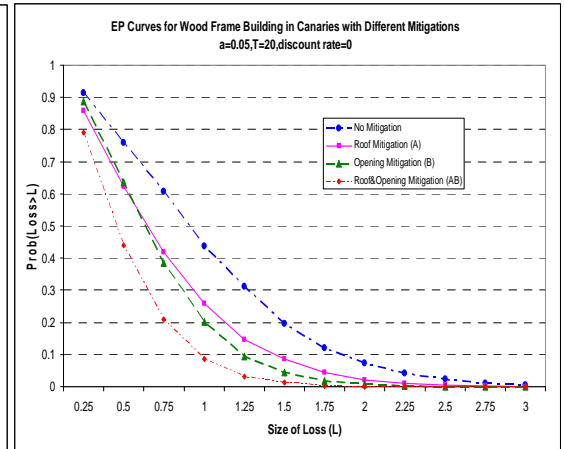


Figure 3.13

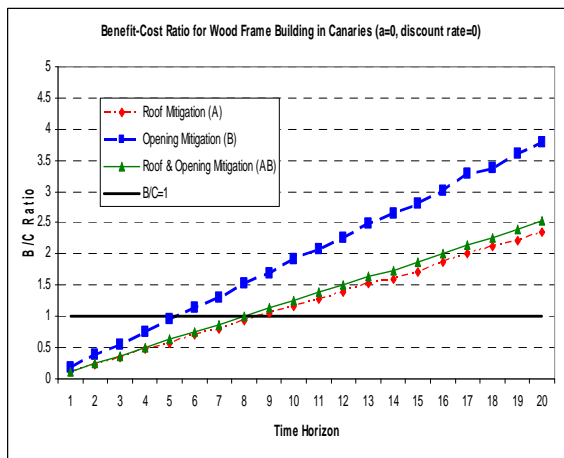


Figure 3.14

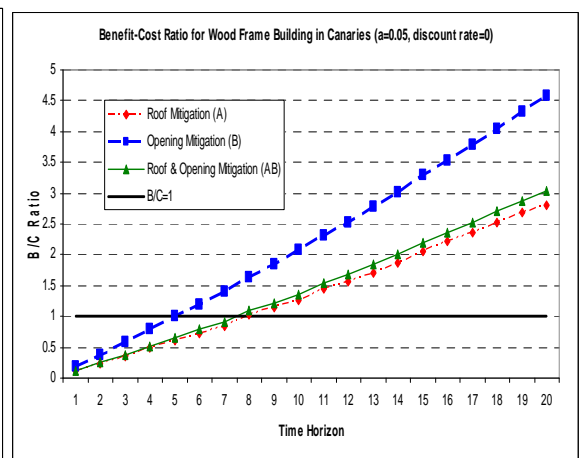


Figure 3.15

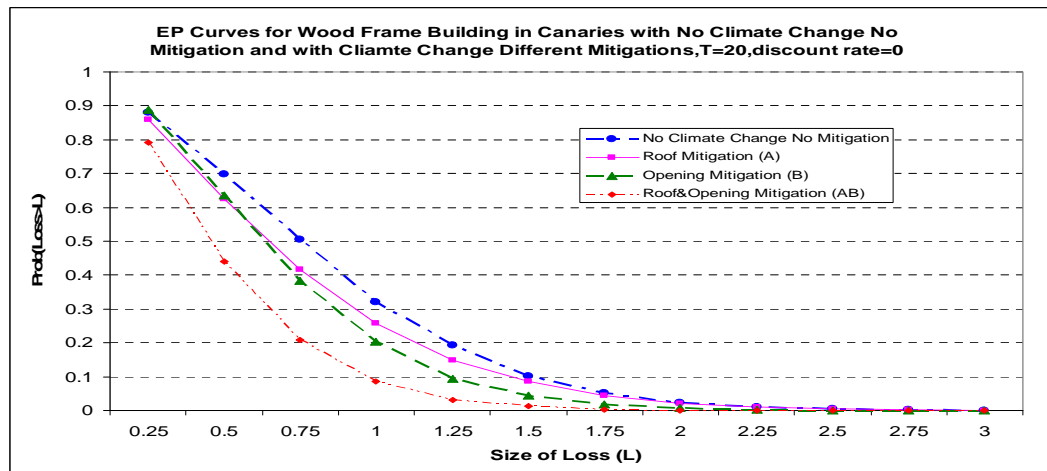


Figure 3.16

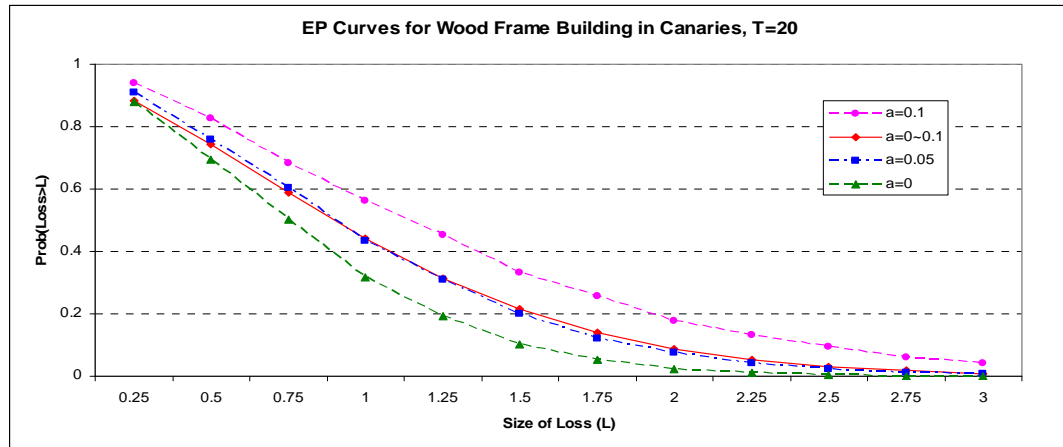


Figure 3.17

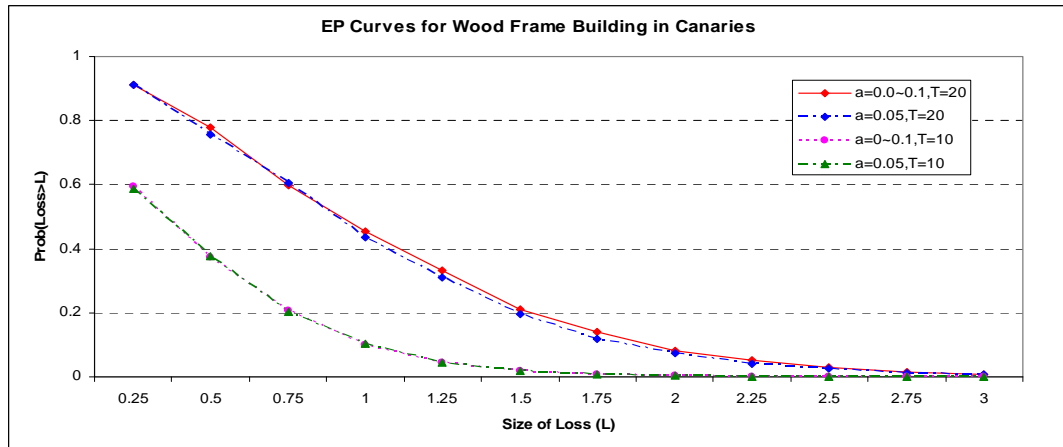


Figure 3.18

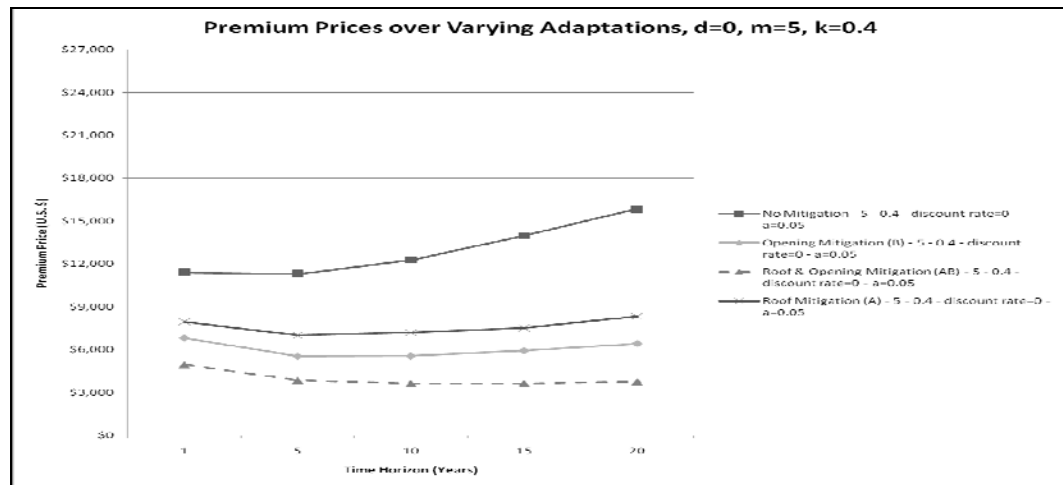


Figure 3.19

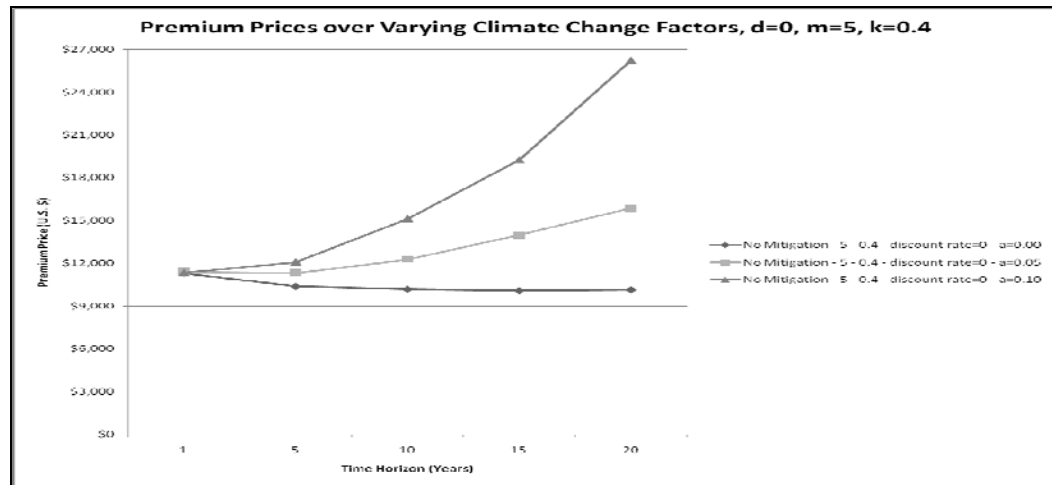


Figure 3.20

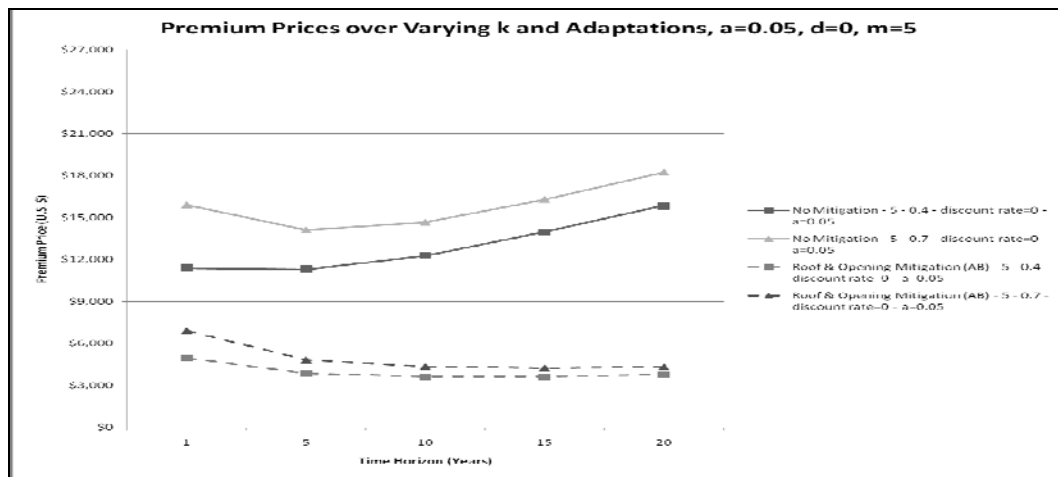


Figure 3.21

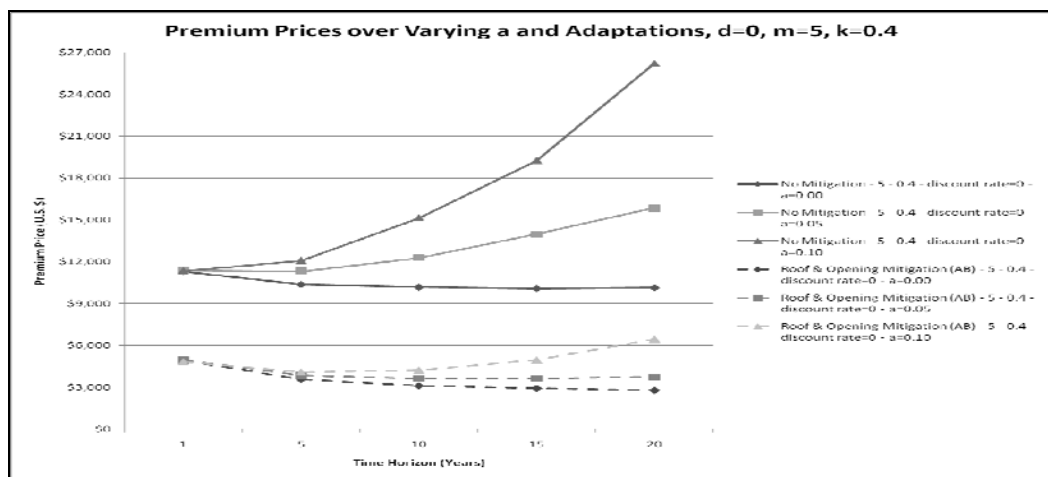


Figure 3.22: A Simple Model that Estimates the Probability and Level of Storm Activity Rate Based on Historical Storm Activity Rate

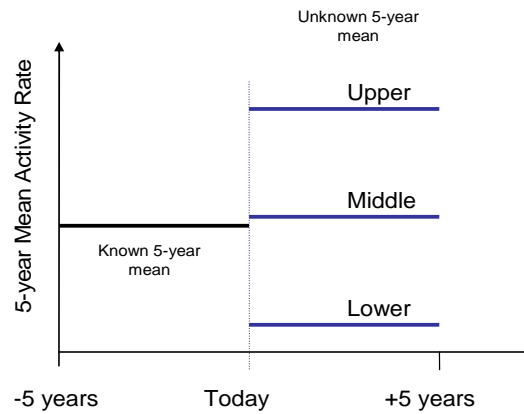


Figure 3.23

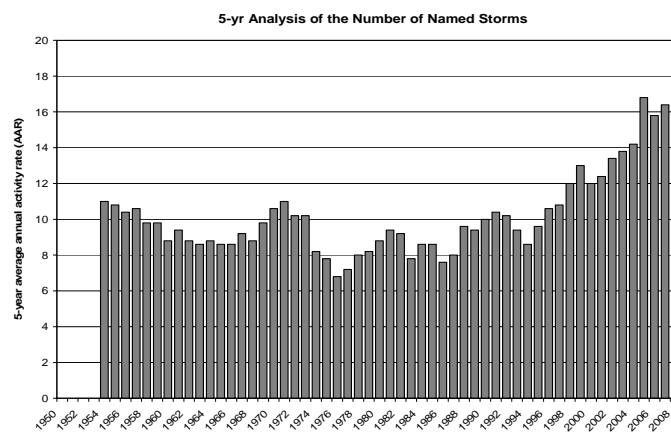


Figure 3.24

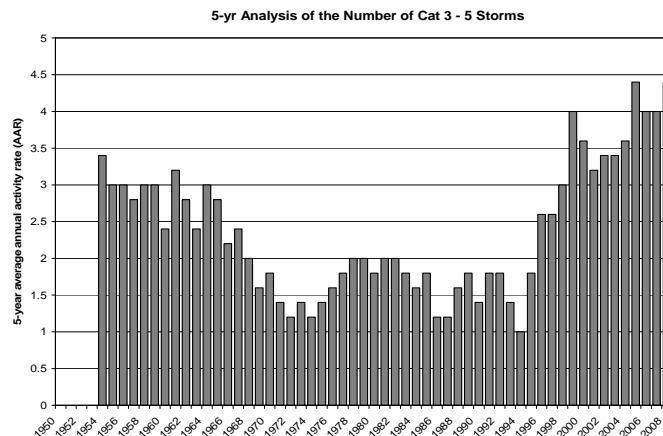


Figure 3.25

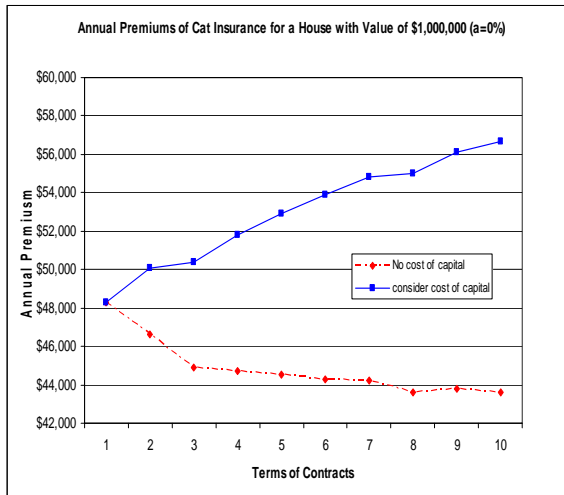


Figure 3.26

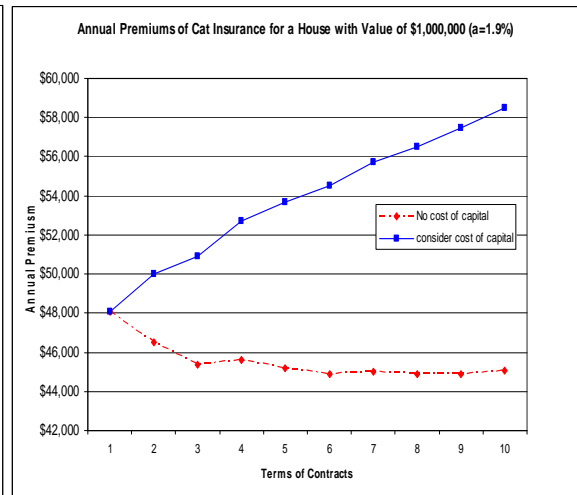
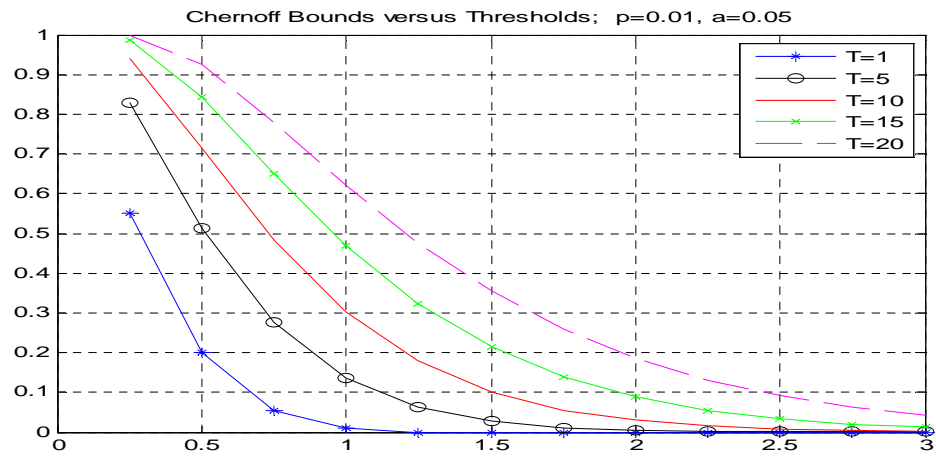


Figure 3.27: Chernoff Bounds versus Thresholds for Various Time Horizons



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